

# Alternating Current

## Introduction to Alternating Current

### 1 Mark Questions

1. Why is the use of AC voltage preferred over DC voltage? Give two reasons. [All India 2014]

**Ans.** The use of AC voltage is preferred over DC voltage because of

(i) the loss of energy in transmitting the AC voltage over long distances with the help of step-up transformers is negligible as compared to DC voltage.

(ii) AC voltage can be stepped up and stepped down as per the requirement by using a transformer.

2. The current flowing through a pure inductance 2mH is,  $i = (15 \cos 300t)$  A. What is the (i) rms and average value of current for a complete cycle? [Foreign 2011]



To calculate different values for AC with the help of given equation, compare the given equation with standard equation of AC.

Current flowing through the inductor,

$$i = 15 \cos 300t$$

Comparing with  $i = i_0 \sin \omega t$

Here, peak value of current,

$$i_0 = 15 \text{ A}$$

(i) For complete cycle, rms value of current

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} \text{ A}$$

(ii) For complete cycle, average value of current is zero

$$\text{i.e.} \quad i_{\text{av}} = 0 \quad (1/2 \times 2 = 1)$$

3. Define the term wattless current. [Delhi 2011]

**Wattless Current** The current in an AC circuit when average power consumption in AC circuit is zero, is referred as wattless current.

If  $\phi$  is the phase difference between voltage and current then power associated with  $i \sin \phi$  component of current is termed as wattless current. (1)

**4. Define the term rms value of the current. How is it related to the peak value? [All India 2010c]**

**Ans.** It is defined as the value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle. It is also called virtual value or effective value of AC.

Let the peak value of the current be  $I_0$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

where,  $I_0$  = peak value of AC.

**5. How much average power over a complete cycle, does an AC source supply to a capacitor? [Delhi 2009c]**

Average power,  $P_{av} = V_{rms} \times I_{rms} \times \cos \phi$

But for pure capacitive circuit,  $\phi = 90^\circ = \frac{\pi}{2}$

$$P_{av} = V_{rms} \times I_{rms} \times \cos 90^\circ = 0$$

$$\therefore P_{av} = 0$$

6. An AC current,  $I = I_0 \sin \omega t$  produces certain heat  $H$  in a resistor  $R$  over a time  $T = 2\pi/\omega$ . Write the value of the DC current that would produce the same heat in the same resistor in the same time.

[All India 2009C]

**Ans.** It is defined as the value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle. It is also called virtual value or effective value of AC.

Let the peak value of the current be  $I_0$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

where,  $I_0$  = peak value of AC.

7. The instantaneous current and voltage of an AC circuit are given by  $I = (10 \sin 300t)$  A and  $V = (200 \sin 300t)$  V. What is the power dissipation in the circuit? [All India 2008]

Given,  $I = 10 \sin 300t$  and  $V = 200 \sin 300t$ .

Comparing with  $I = I_0 \sin \omega t$  and  $V = V_0 \sin \omega t$ , we get

$$I_0 = 10 \text{ A}, V_0 = 200 \text{ V}$$

$$\Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} \text{ A}, V_{rms} = \frac{200}{\sqrt{2}} \text{ V}$$

Phase difference,  $\phi = 0^\circ$

$\therefore$  Power dissipated,

$$P_{av} = V_{rms} \times I_{rms} \times \cos \phi$$

$$= \left( \frac{10}{\sqrt{2}} \right) \left( \frac{200}{\sqrt{2}} \right) \cos 0^\circ$$

$$P_{av} = 1000 \text{ W} \quad (1)$$

8. The instantaneous current and voltage of an AC circuit are given by  $I = (10 \sin 314t)$  A and  $V = (50 \sin 314t)$  V. What is the power dissipation in the circuit? [All India 2008]

Given,  $I = 10 \sin 300t$  and  $V = 200 \sin 300t$ .

Comparing with  $I = I_0 \sin \omega t$  and  $V = V_0 \sin \omega t$ , we get

$$I_0 = 10 \text{ A}, V_0 = 200 \text{ V}$$

$$\Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} \text{ A}, V_{rms} = \frac{200}{\sqrt{2}} \text{ V}$$

Phase difference,  $\phi = 0^\circ$

$\therefore$  Power dissipated,

$$P_{av} = V_{rms} \times I_{rms} \times \cos \phi$$

$$= \left( \frac{10}{\sqrt{2}} \right) \left( \frac{200}{\sqrt{2}} \right) \cos 0^\circ$$

$$P_{av} = 1000 \text{ W} \quad (1)$$

9. The instantaneous current and voltage of an AC circuit are given by

$$I = (10 \sin 314t) \text{ A and}$$

$$V = 50 \sin \left( 314t + \frac{\pi}{2} \right) \text{ V. What is the}$$

power dissipation in the circuit?

[All India 2008,

Given,  $I = 10 \sin 314t$

$$V = 50 \sin \left( 314t + \frac{\pi}{2} \right)$$

Comparing with  $I = I_0 \sin \omega t$

and  $V = V_0 \sin (\omega t + \phi)$

$$I_0 = 10 \text{ A, } V_0 = 50 \text{ V}$$

$$\Rightarrow I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ A}$$

Phase difference,  $\phi = \frac{\pi}{2}$

$$V_{\text{rms}} = \frac{50}{\sqrt{2}} \text{ V}$$

$$\begin{aligned} \therefore P_{\text{av}} &= V_{\text{rms}} I_{\text{rms}} \cos \phi \\ &= \frac{10}{\sqrt{2}} \times \frac{50}{\sqrt{2}} \times \cos \frac{\pi}{2} = 0 \end{aligned}$$

$$\therefore \cos \frac{\pi}{2} = 0$$

## 2 Marks Questions

10. A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate

- the resistance of the bulb
- the rms current through the bulb. [All India 2012]

(i)  $P = 150 \text{ W}, V = 220 \text{ V}$

Resistance of the bulb,

$$R = \frac{V^2}{P} \quad (1/2)$$

$$R = \frac{220 \times 220}{150} = 322.7 \, \Omega \quad (1/2)$$

(ii) As,  $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220}{322.7}$  ( $V_{\text{rms}} = V = 220 \text{ V}$ ) (1/2)

$$\Rightarrow I_{\text{rms}} = 0.68 \text{ A} \quad (1/2)$$

11. Prove that an ideal capacitor in an AC circuit does not dissipate power. [Delhi 2011]

Since, average power consumption in an AC circuit is given by

$$P_{\text{av}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos \phi \quad (1)$$

But in pure capacitive circuit, phase difference between voltage and current is given by

$$\phi = \frac{\pi}{2}$$

$$\therefore P_{\text{av}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos \frac{\pi}{2}$$

$$\Rightarrow P_{\text{av}} = 0 \quad \left( \because \cos \frac{\pi}{2} = 0 \right)$$

Thus, no power is consumed in pure capacitive AC circuit. (1)

12. (i) For a given AC,  $i = i_m \sin \omega t$ , show that the average power dissipated in a resistor  $R$  over a complete cycle is  $\frac{1}{2} i_m^2 R$ .

(ii) A light bulb is rated at 100 W for a 220 V AC supply. Calculate the resistance of the bulb. [All India 2013]

(i) The average power dissipated,

$$\bar{P} = (i^2 R) = (i_m^2 R \sin^2 \omega t) = i_m^2 R (\sin^2 \omega t)$$

$$\therefore \sin^2 \omega t = \frac{1}{2} (1 - \cos 2\omega t)$$

$$\therefore (\sin^2 \omega t) = \frac{1}{2} (1 - (\cos 2\omega t)) = \frac{1}{2} \quad (\because (\cos 2\omega t) = 0)$$

$$\therefore \bar{P} = \frac{1}{2} i_m^2 R \quad (2)$$

(ii) Power of the bulb,  $P = 100$  W and voltage,  $V = 220$  V

The resistance of the bulb is given as

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484 \, \Omega \quad (1)$$

13. (i) When an AC source is connected to an ideal capacitor, then show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a DC and then AC source. What happens in each case if the capacitance of the capacitor is reduced? [Delhi 2013C]

(i) When a source of AC is connected to a capacitor of capacitance  $C$ , the charge on it grows from zero to maximum steady value  $Q_0$ .

The energy stored in a capacitor is

$$E = \frac{1}{2} C V_0^2$$

where,  $V_0$  is maximum potential difference across the plates of the capacitor.

The alternating voltage applied is

$$E = E_0 \sin \omega t$$

and the current leads the emf by a phase angle of  $\pi/2$ .

$$E = E_0 \sin \omega t$$

and the current leads the emf by a phase angle of  $\pi/2$ .

$$I = I_0 \sin \left( \omega t + \frac{\pi}{2} \right)$$

$$= I_0 \cos \omega t$$

$\therefore$  Work done over a complete cycle is

$$\begin{aligned} W &= \int_0^T E I dt = \int_0^T (E_0 \sin \omega t)(I_0 \cos \omega t) dt \\ &= \frac{E_0 I_0}{2} \int_0^T 2 \sin \omega t \cos \omega t dt \\ W &= \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t dt \\ &= \frac{E_0 I_0}{2} \left[ 1 - \frac{\cos 2\omega t}{2\omega} \right]_0^T = 0 \quad \left( 1 \frac{1}{2} \right) \end{aligned}$$

- (ii) When DC source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When AC source is connected, the capacitor offers capacitive reactance  $X_C = \frac{1}{\omega C}$ . The current flows in the circuit

and the lamp glows. On reducing  $C$ ,  $X_C$  increases. Therefore, the glowing of the bulb reduces. ( 1 )

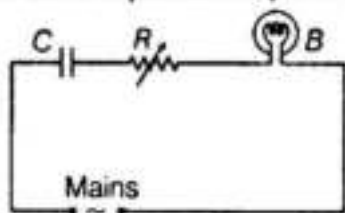
1. A reactive element in an AC circuit causes the current flowing
- (i) to lead in phase by  $\pi/2$
  - (ii) to lag in phase by  $\pi/2$
- w.r.t. the applied voltage. Identify the element in each case. [Delhi 2010C]

Ans

- (i) In case of pure capacitive circuit, the current leads in phase by  $\pi/2$  with respect to the applied voltage. So, the element will be a capacitor. (1/2)
- (ii) In case of pure inductive circuit, the current lags in phase by  $\pi/2$  with respect to the applied voltage. So, the element will be an inductor. (1/2)

## 2 Marks Questions

2. A capacitor  $C$ , a variable resistor  $R$  and a bulb  $B$  are connected in series to the AC mains in the circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor keeping resistance  $R$  to be the same (ii) the resistance  $R$  is increased keeping the same capacitance? [Delhi 2014]



Ans.

- (i) As, the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease, i.e.  $V = \frac{Q}{C}$ . As a result, the potential drop across the bulb will increase as they are connected in series. Thus, its brightness will increase. (1)
- (ii) As the resistance  $R$  is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease as they are connected in series. Thus, its brightness will decrease. (1)

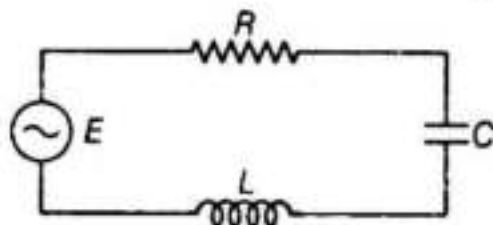


3. The figure shows a series  $L$ - $C$ - $R$  circuit connected to a variable frequency 200 V source with  $L = 50 \text{ mH}$ ,  $C = 80 \text{ } \mu\text{F}$  and  $R = 40 \text{ } \Omega$ .

Determine

- (i) the source frequency which derives the circuit in resonance.
- (ii) the quality factor ( $Q$ ) of the circuit.

[All India 2014C]



Ans.

Given,  $L = 50 \text{ mH} = 50 \times 10^{-3} \text{ H}$

$$C = 80 \text{ } \mu\text{F} = 80 \times 10^{-6} \text{ F}$$

$$R = 40 \text{ } \Omega, V = 200 \text{ V}$$

- (i) In the  $L$ - $C$ - $R$ , the resonant angular frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 80 \times 10^{-6}}} \\ = 500 \text{ rad/s}$$

$\therefore$  The actual/source frequency is given by

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$$

$$\Rightarrow \nu = \frac{500}{2\pi} = \frac{250}{\pi} = 79.61 \approx 80 \text{ Hz}$$

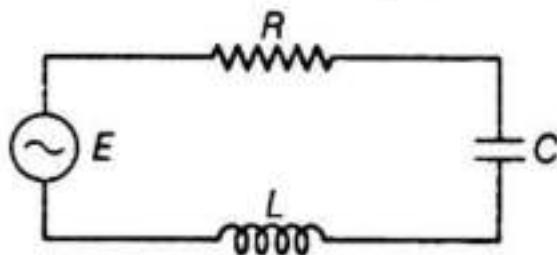
- (ii) Quality factor,  $Q = \frac{\omega_0 L}{R}$

$$= \frac{500 \times 50 \times 10^{-3}}{40} = 0.625 \\ \text{(1/2)}$$

4. The figure shows a series  $L$ - $C$ - $R$  circuit connected to a variable frequency 250 V source with  $L = 40\text{ mH}$ ,  $C = 100\text{ }\mu\text{F}$  and  $R = 50\text{ }\Omega$ .

Determine

- (i) the source frequency which derives the circuit in resonance,
  - (ii) the quality factor ( $Q$ ) of the circuit.
- [All India 2014]



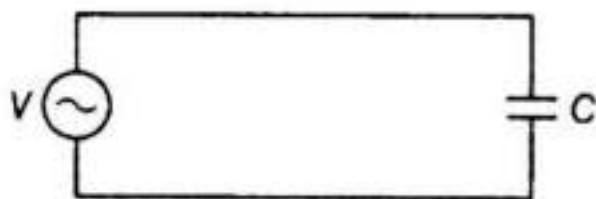
Ans. Refer to ans3

5. The figure shows a series  $L$ - $C$ - $R$  circuit connected to a variable frequency 220 V source with  $L = 80\text{ mH}$ ,  $C = 50\text{ }\mu\text{F}$  and  $R = 60\text{ }\Omega$ .

Determine

- (i) the source frequency which derives the circuit in resonance.
- (ii) the quality factor  $Q$  of the circuit.

[All India 2014]

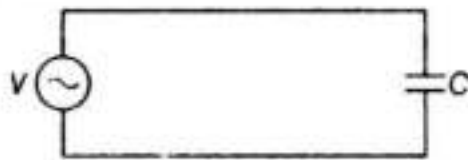


Ans Refer to ans3.

6. Show that the current leads the voltage in phase by  $\pi/2$  in an AC circuit containing an ideal capacitor.
- [Foreign 2014]

Ans.

Let us consider a capacitor  $C$  connected to an AC source as shown below:



Let the AC voltage applied be

$$V = V_m \sin \omega t$$

$$\therefore V = \frac{q}{C}$$

Applying Kirchhoff's loop rule, we have

$$V_m \sin \omega t = \frac{q}{C} \Rightarrow q = CV_m \sin \omega t$$

$$\text{Also, } i = \frac{dq}{dt}$$

$$\Rightarrow i = \frac{d}{dt} (V_m \sin \omega t)$$

$$i = \omega CV_m \cos \omega t \quad \dots (i)$$

$$\text{We know that } \cos \omega t = \sin \omega t + \pi/2 \quad \dots (ii)$$

$$\text{In the circuit,} \quad (1)$$

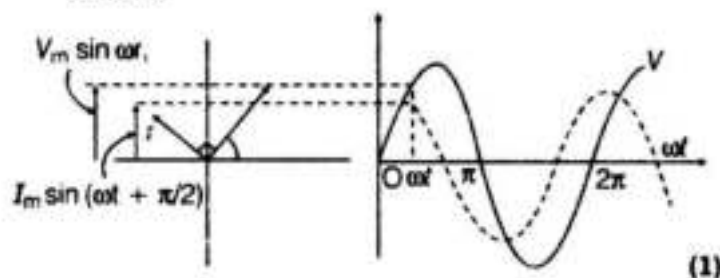
$$V_m = i_m X_C = i_m \frac{1}{\omega C}$$

$$\Rightarrow i_m = V_m \omega C \quad \dots (iii)$$

Substituting the value of Eqs. (ii) and (iii) in Eq. (i) we get

$$i = i_m \sin \left( \omega t + \frac{\pi}{2} \right)$$

The phase diagram which shows the current lead the voltage in phase by  $90^\circ$  is given below:



(1)

7. In a series L-C-R circuit, obtain the conditions under which (i) the impedance of circuit is minimum and (ii) wattless current flows in the circuit. [Foreign 2014]

Ans.

- (i) The impedance of a series L-C-R circuit is

$$\text{given by } Z = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2}$$

Z will be minimum when  $\omega L = \frac{1}{\omega C}$ , i.e.

When the circuit is under resonance  
Hence, or this condition Z will be minimum and equal to R. (1)

- (ii) Average power dissipated through a series L-C-R circuit is given by

$$P_{av} = E_V I_V \cos \phi$$

where,  $E_V$  = rms value of alternating voltage

$I_V$  = rms value of alternating current

$\phi$  = phase difference between current and voltage

For wattless current, the power dissipated through the circuit should be zero.

$$\text{i.e. } \cos \phi = 0 \Rightarrow \cos \phi = \cos \frac{\pi}{2}$$

$$\Rightarrow \phi = \frac{\pi}{2}$$

Hence, the condition for wattless current is that the phase difference between the current and the circuit is purely inductive or purely capacitive. **(1)**

- 8.** Calculate the quality factor of a series  $L$ - $C$ - $R$  circuit with  $L = 2.0$  H,  $C = 2\mu\text{F}$  and  $R = 10\ \Omega$ . Mention the significance of quality factor in  $L$ - $C$ - $R$  circuit. **[Foreign 2012]**

**Ans.**

Given,  $L = 2.0$  H

$$C = 2\mu\text{F} \\ = 2 \times 10^{-6} \text{ F}$$

$$R = 10\ \Omega$$

$$\begin{aligned} \text{Now, } Q\text{-factor} &= \frac{1}{R} \sqrt{\frac{L}{C}} \\ &= \frac{1}{10} \sqrt{\frac{2}{2 \times 10^{-6}}} = \frac{1}{10 \times 10^{-3}} \\ &= \frac{1}{10^{-2}} = 100 \end{aligned} \quad \textbf{(1)}$$

Quality factor is also defined as

$$Q = 2\pi f \times \frac{\text{Energy stored}}{\text{Power loss}}$$

- 9.** An alternating voltage given by  $V = 140 \sin 314t$  is connected across a pure resistor of  $50\ \Omega$ . Find

- (i) the frequency of the source.
- (ii) the rms current through the resistor.

**[All India 2012]**

9. An alternating voltage given by  $V = 140 \sin 314t$  is connected across a pure resistor of  $50 \Omega$ . Find
- (i) the frequency of the source.
  - (ii) the rms current through the resistor.
- [All India 2012]

Ans.

Given,  $V = 140 \sin 314t$ ,  $R = 50 \Omega$

Comparing it with  $V = V_0 \sin \omega t$

(i) Here,  $\omega = 314 \text{ rad/s}$  (1/2)

i.e.  $2\pi\nu = 314$  [ $\because \omega = 2\pi\nu$ ]

$$\Rightarrow \nu = \frac{314}{2\pi}$$

$$\nu = \frac{31400}{2 \times 314}$$

$$= 50 \text{ Hz}$$

Frequency of AC,  $\nu = 50 \text{ Hz}$  (1/2)

(ii) As,  $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$  and  $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$  (1/2)

Here,  $V_0 = 140 \text{ V}$

$$\Rightarrow V_{\text{rms}} = \frac{140}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 70\sqrt{2} \text{ V}$$

$$\therefore I_{\text{rms}} = \frac{70\sqrt{2}}{R} = \frac{70\sqrt{2}}{50}$$

$$= 1.9 \text{ A or } 2 \text{ A} \quad (1/2)$$

**10.** An alternating voltage given by  $V = 280 \sin 50 \pi t$  is connected across a pure resistor of  $40 \Omega$ . Find

(i) the frequency of the source.

(ii) the rms current through the resistor.

**[All India 2012]**

**Ans.**

Given,  $V = 280 \sin 50 \pi t$ ,  $R = 40 \Omega$

Comparing it with standard equation,

$$V = V_0 \sin \omega t$$

$$V_0 = 280 \text{ V}$$

$$\omega = 50 \pi \text{ rad/s}$$

(i) As,  $\omega = 2\pi\nu = 50 \pi$  (1/2)

Frequency of AC,  $\nu = \frac{50}{2\pi} \pi = 25 \text{ Hz}$  (1/2)

(ii)  $V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{280}{\sqrt{2}} \text{ V}$

$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{280}{\sqrt{2} \times 40}$

$$I_{\text{rms}} = \frac{280}{1.414 \times 40} = 4.95 \text{ A}$$

(1/2  $\times$  2 = 1)

- 11.** An alternating voltage given by  $V = 70 \sin 100 \pi t$  is connected across a pure resistor of  $25 \Omega$ . Find
- (i) the frequency of the source.
  - (ii) the rms current through the resistor.
- [All India 2012]**

**Ans.**

Given,  $V = 70 \sin 100 \pi t$

On comparing with standard equation,

$$V = V_0 \sin \omega t \Rightarrow V_0 = 70 \text{ V}$$

$$\omega = 100 \pi \text{ rad/s}$$

(i) As,  $\omega = 2\pi\nu = 100 \pi$

Frequency of the source,

$$\nu = 50 \text{ Hz}$$

$$(1/2 \times 2 = 1)$$

(ii)  $V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{70}{\sqrt{2}}$

$$\begin{aligned} \therefore I_{\text{rms}} &= \frac{V_{\text{rms}}}{R} \\ &= \frac{(70/\sqrt{2})}{25} = \frac{70}{1.414 \times 25} \\ &= 1.98 \text{ A} \end{aligned}$$

$$(1)$$



12. (i) An alternating voltage  $V = V_m \sin \omega t$  applied to a series  $L-C-R$  circuit derives a current given by  $I = I_m \sin (\omega t + \phi)$ . Deduce an expression for the average power dissipated over a cycle.
- (ii) For circuit used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

[Foreign 2011; Delhi 2009]

Ans.

- (i) Let at any instant, the current and voltage in an  $L-C-R$  series AC circuit is given by

$$I = I_0 \sin \omega t$$

$$V = V_0 \sin (\omega t + \phi)$$

The instantaneous power is given by

$$P = VI = V_0 \sin (\omega t + \phi) I_0 \sin \omega t$$

$$P = \frac{V_0 I_0}{2} [2 \sin \omega t \sin (\omega t + \phi)]$$

$$P = VI = \frac{V_0 I_0}{2} [\cos \phi - \cos (2\omega t + \phi)] \dots (i)$$

$$[\because 2 \sin A \sin B = \cos (A - B) - \cos (A + B)]$$

Work done for a very small time interval  $dt$  is given by

$$dW = P dt$$

$$dW = VI dt$$

$\therefore$  Total work done over  $T$ , a complete cycle is given by

$$W = \int_0^T VI dt \quad (1/2)$$

$$\text{But, } P_{av} = \frac{W}{T} = \frac{\int_0^T VI dt}{T}$$

$$\Rightarrow P_{av} = \frac{1}{T} \int_0^T VI dt$$

$$= \frac{1}{T} \int_0^T \frac{V_0 I_0}{2} [\cos \phi - \cos (2\omega t + \phi)] dt$$

$$\text{or } P_{av} = \frac{V_0 I_0}{2T} \left[ \int_0^T \cos \phi \, dt - \int_0^T \cos (2\omega t + \phi) \, dt \right]$$

$$= \frac{V_0 I_0}{2T} [\cos \phi [t]_0^T - 0]$$

(By trigonometry)

$$\text{or } P_{av} = \frac{V_0 I_0}{2T} \times \cos \phi \times T = \frac{V_0 I_0}{2} \times \cos \phi$$

$$P_{av} = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos \phi$$

$$\Rightarrow P_{av} = V_{rms} \times I_{rms} \times \cos \phi \quad (1/2)$$

This is required expression.

$$(ii) \because \text{Power factor, } \cos \phi = \frac{R}{Z}$$

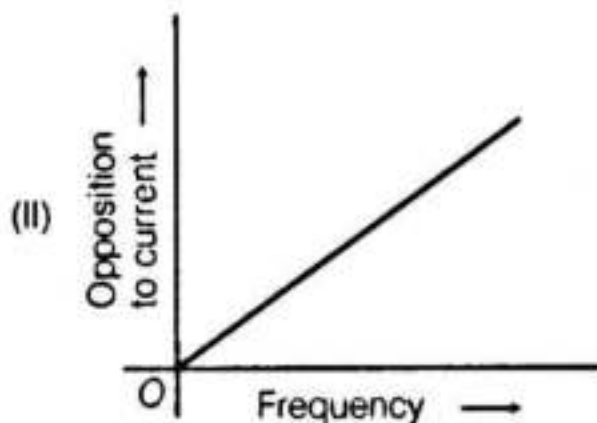
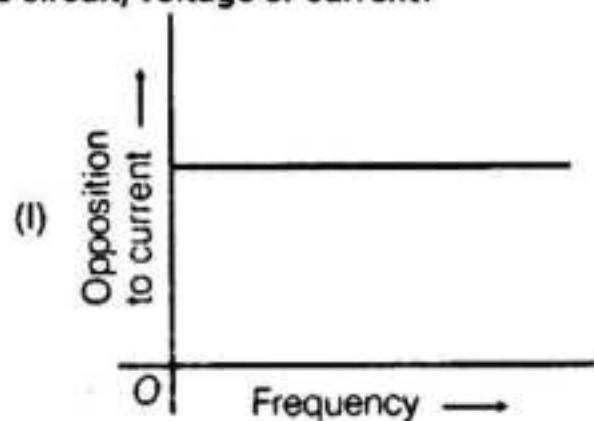
where,  $R$  = resistance and  $Z$  = impedance

Low power factor ( $\cos \phi$ ) implies lower ohmic resistance and higher power loss as

$$P_{av} \propto \frac{1}{R} \text{ in power system (transmission line)} \quad (1)$$

- 13.(i) The graphs (I) and (II) represent the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit element corresponding to each graph.
- (ii) Write the expression for the impedance offered by the series combination of the above two elements connected across the AC sources. Which will be ahead in phase in

this circuit, voltage or current?



[All India 2011C]

Ans.

- (i) From graph (I), it is clear that resistance (opposition to current) is not changing with frequency, i.e. resistance does not depend on frequency of applied source, so the circuit element here is pure resistance ( $R$ ). From graph (II), it is clear that resistance increases linearly with frequency, so the circuit element here is an inductor.

Inductive resistance,  $X_L = 2\pi fL$

$$\Rightarrow X_L \propto f \quad (1)$$

- (ii) Impedance offered by the series combination of resistance  $R$  and inductor  $L$ .

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

In  $L$ - $R$  circuit, the applied voltage leads the current in phase by  $\frac{\pi}{2}$ .

(1)

14. An AC source of voltage  $V = V_m \sin \omega t$  is applied across a series L-C-R. Draw the phasor diagram for this circuit when the

- capacitive impedance exceeds the inductive impedance.
- inductive impedance exceeds the capacitive impedance. [All India 2008C]

Ans.

- (i) Capacitive impedance exceeds the inductive impedance.

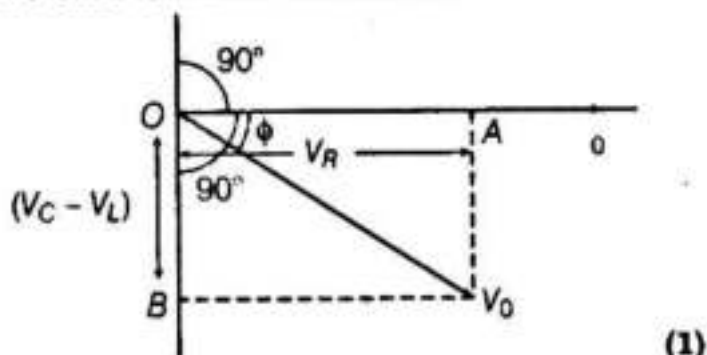
i.e.  $X_C > X_L$  or  $V_C > V_L$

$$\left[ \because X_C = \frac{V_C}{I} \text{ and } X_L = \frac{V_L}{I} \right]$$

Phase difference in an L-C-R circuit

$\tan \phi = \frac{X_L - X_C}{R}$ . Here,  $X_C > X_L$ , so  $\tan \phi$  will

be negative. Hence, voltage lags behind the current by a phase angle  $\phi$ . In this case, AC circuit is called as **capacitance dominated circuit**. The phasor diagram in this case is shown as below:

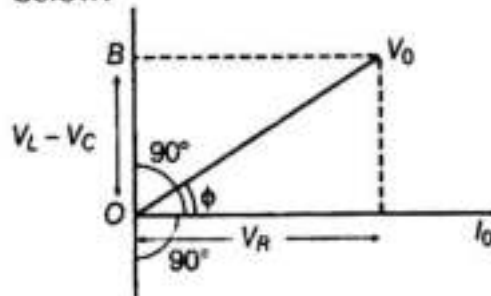


- (ii) Inductive impedance exceeds the capacitive impedance, i.e.  $X_L > X_C$  or  $V_L > V_C$ .

$$\text{Phase difference, } \tan \phi = \frac{X_L - X_C}{R}$$

Here,  $X_L > X_C$ , so  $\tan \phi$  is positive. Hence, voltage leads the current by a phase angle  $\phi$ . In this case, AC circuit is called as **inductance dominated circuit**.

The phasor diagram in this case is shown as below:



(1)

15. An AC source of voltage  $V = V_m \sin \omega t$  is applied across a

- series R-C circuit in which the capacitive impedance is a times the resistance in the circuit.
- series R - L circuit in which the inductive impedance is b times the resistance in the circuit.

Calculate the value of the power factor of the circuit in each case. [All India 2008C]

Ans.



Power factor is a ratio of the pure resistance to the total impedance of an AC circuit. So, to determine the power factor, first of all total impedance is to be calculated

$$\therefore \text{Power factor, } \cos \phi = \frac{R}{Z}$$

where,  $\phi$  = Phase difference between V and Z

R = Ohmic resistance

Z = Impedance.

(i) Given,  $X_C = aR$

$$\begin{aligned} \therefore \text{Impedance, } Z &= \sqrt{R^2 + X_C^2} \\ &= \sqrt{R^2 + a^2 R^2} = R\sqrt{1 + a^2} \end{aligned}$$

$$\therefore \cos \phi = \frac{R}{R\sqrt{1 + a^2}} = \frac{1}{\sqrt{1 + a^2}}$$

$$\text{Power factor} = \frac{1}{\sqrt{1 + a^2}} \quad (1)$$

(ii) Given,  $X_L = bR$

$$\begin{aligned} \therefore Z &= \sqrt{R^2 + X_L^2} \\ &= \sqrt{R^2 + b^2 R^2} \end{aligned}$$

$$Z = R \sqrt{1 + b^2}$$

$$\therefore \text{Power factor, } \cos \phi = \frac{R}{Z}$$

$$= \frac{R}{R \sqrt{1 + b^2}} = \frac{1}{\sqrt{1 + b^2}}$$

$$\therefore \text{Power factor} = \frac{1}{\sqrt{1 + b^2}} \quad (1)$$

### 3 Marks Questions

16. A voltage  $V = V_0 \sin \omega t$  is applied to a series L-C-R Derive the expression for the average power dissipated over a cycle.

Under what condition is (i) no power dissipated even though the current flows through the circuit (ii) maximum power dissipated in the circuit? [All India 2014]

Ans.

Let applied alternating voltage,

$$V = V_0 \sin \omega t \quad \dots (i)$$

We know that,

$$I = I_0 \sin (\omega t - \phi) \quad \dots (ii)$$

$$\text{Power, } P = VI \quad (1/2)$$

Substituting the value of  $V$  and  $I$  from Eq. (i) and Eq. (ii), we get

$$P = V_0 \sin \omega t \cdot I_0 \sin (\omega t - \phi)$$

So, the instantaneous power is given by

$$\therefore \sin (\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$$

(i) If  $\phi = 90^\circ$ , then no power is dissipated even though the current flows through the circuit.

$$P_{av} = 0$$

$$\therefore \tan \phi = \left( \frac{X_L - X_C}{R} \right)$$

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = \infty \quad (\because \tan 90^\circ = \infty) \quad (1/2)$$

(ii) If  $\phi = 0^\circ$ , then maximum power is dissipated in the circuit.

$$P_{av} =$$

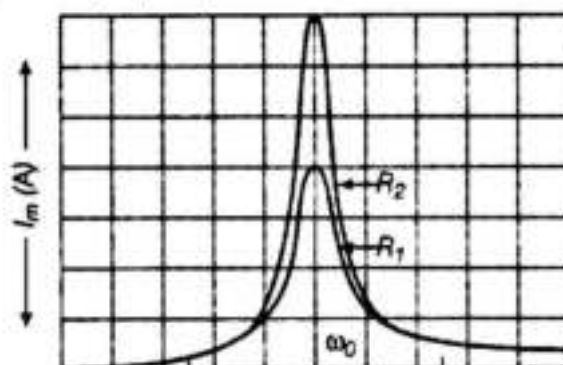
$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0 \quad (\because \tan 0^\circ = 0)$$

$$\Rightarrow X_L = X_C \quad (\text{Resonance}) \quad (1/2)$$

17. If a series  $L$ - $C$ - $R$  circuit connected to an AC source of variable frequency and voltage  $V = V_m \sin \omega t$ , draw a plot showing the variation of current  $I$  with angular frequency  $\omega$  for two different values of resistances  $R_1$  and  $R_2$  ( $R_1 > R_2$ ). Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance. [Delhi 2013]

Ans.

Figure shows the variation of  $i_m$  with  $\omega$  in a  $L$ - $C$ - $R$  series circuit for two values of resistance  $R_1$  and  $R_2$  ( $R_1 > R_2$ ).





The condition for resonance in the  $L$ - $C$ - $R$  circuit is  $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

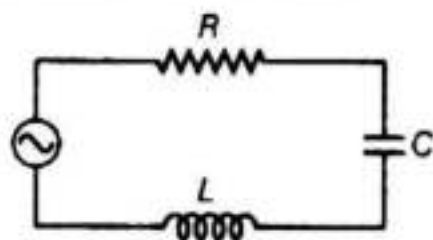
We see that the current amplitude is maximum at the resonant frequency  $\omega_0$ . Since,  $i_m = V_m/R$  at resonance, the current amplitude for case  $R_2$  is sharper to that for case  $R_1$ . (1)

Quality factor or simply the  $Q$ -factor of a resonant  $L$ - $C$ - $R$  circuit is defined as the ratio of voltage drop across the capacitor (or inductor) to that of applied voltage.

It is given by  $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$

The  $Q$ -factor determines the sharpness of the resonance curve and if the resonance is less sharp, not only is the maximum current less, the circuit is close to the resonance for a larger range  $\Delta\omega$  of frequencies and the tuning of the circuit will not be good. So, less sharp the resonance, less is the selectivity of the circuit while higher is the  $Q$ , sharper is the resonance curve and lesser will be the loss in energy of the circuit. (1)

18. The figure shows a series  $L$ - $C$ - $R$  circuit with  $L = 10.0 \text{ H}$ ,  $C = 40 \mu\text{F}$ ,  $R = 60 \Omega$  connected to a variable frequency  $240 \text{ V}$  source. Calculate



- (i) the angular frequency of the source which derives the circuit at resonance.
- (ii) the current at the resonating frequency.
- (iii) the rms potential drop across the inductor at resonance. [Delhi 2012]

Ans.

Given,  $L = 10 \text{ H}$ ,  $C = 40 \mu\text{F}$ ,

$R = 60 \Omega$ ,  $V_{\text{rms}} = 240 \text{ V}$

- (i) Resonating angular frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}}$$

$$\omega_0 = \frac{1}{20 \times 10^{-3}} = 50 = 50 \text{ rad/s} \quad (1)$$

- (ii) Current at resonating frequency,

$$\begin{aligned} I_{\text{rms}} &= \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} \quad (\because \text{At resonance } Z = R) \\ &= \frac{240}{60} = 4 \text{ A} = 4 \text{ A} \end{aligned} \quad (1)$$

- (iii)  $\therefore$  Inductive reactance,  $X_L = \omega L$

At resonance,

$$X_L = \omega_0 L = 50 \times 10 = 500 \Omega$$

Potential drop across to inductor,

$$\begin{aligned} V_{\text{rms}} &= I_{\text{rms}} \times X_L = 4 \times 500 \\ &= 2000 \text{ V} \end{aligned} \quad (1)$$

19. A series L-C-R circuit is connected to an AC source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation. [All India 2012]

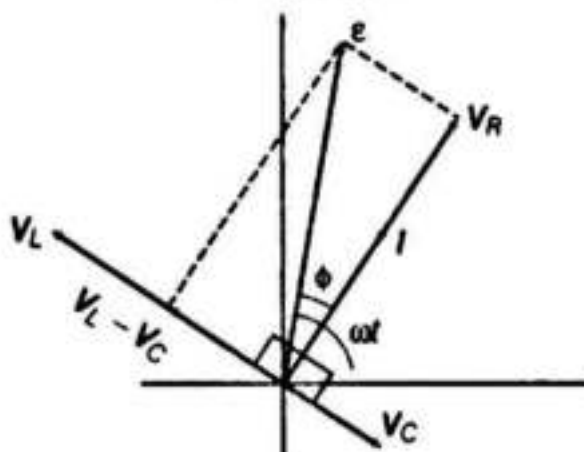
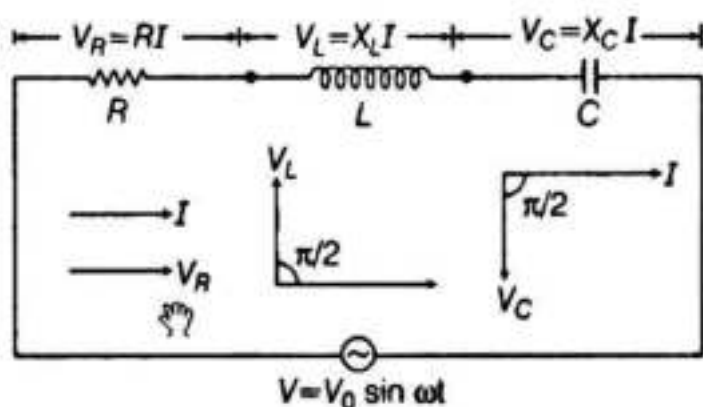
Ans.

Assuming  $X_L > X_C$ ,

$\Rightarrow V_L > V_C$

$\therefore$  Net voltage,  $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

where,  $V_L$ ,  $V_C$  and  $V_R$  are PD across  $L$ ,  $C$  and  $R$  respectively.



(2)

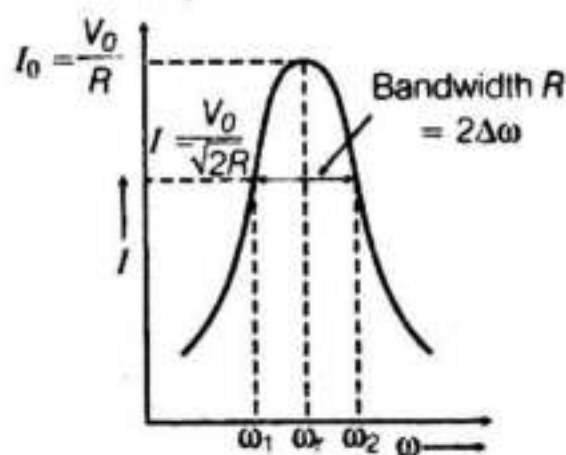
But,  $V_R = IR$ ,  $V_L = IX_L$ ,  $V_C = IX_C$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance of L-C-R circuit,

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2} \quad (1)$$



**20.** A series L-C-R circuit is connected to a 220 V variable frequency AC supply. If  $L = 20 \text{ mH}$ ,  $C = (800/\pi^2) \mu\text{F}$  and  $R = 110 \Omega$ .

- (i) Find the frequency of the source for which average power absorbed by the circuit is maximum.
- (ii) Calculate the value of maximum current amplitude. [Delhi 2010C]

Ans.

Given,  $V_{\text{rms}} = 220 \text{ V}$ ,  $L = 20 \text{ mH} = 2 \times 10^{-2} \text{ H}$ ,

$$R = 110 \Omega,$$

$$C = \frac{800}{\pi^2} \mu\text{F} = \frac{800}{\pi^2} \times 10^{-6} \text{ F}$$

(i) Average power observed by  $L$ - $C$ - $R$  series AC circuit is maximum when circuit is in resonance.

$\therefore$  Resonant frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow = \frac{1}{2\pi\sqrt{2 \times 10^{-2} \times \frac{800}{\pi^2} \times 10^{-6}}}$$

$$\nu_0 = \frac{1000}{2 \times 4} = 125 \text{ s}^{-1}$$

$$\nu_0 = 125 \text{ s}^{-1} \quad \left(1 \frac{1}{2}\right)$$

$$(ii) \text{ As, } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{110} = 2 \text{ A}$$

$$Z = R = 110 \Omega$$

$$I_{\text{rms}} = 2 \text{ A}$$

$\therefore$  Maximum current amplitude,

$$I_0 = I_{\text{rms}} \sqrt{2} = 2 \sqrt{2} \text{ A} \quad \left(1 \frac{1}{2}\right)$$

21. An AC voltage  $V = V_0 \sin \omega t$  is applied across a pure inductor. Obtain an expression for the current / in the circuit and hence obtain the

- inductive reactance of the circuit and
- the phase of the current flowing with respect to the applied voltage. [All India 2010C]

Ans.



Due to change in flux, the emf is induced in the coil. The rate of change of flux will give the value of emf.

Let an alternating voltage,  $V = V_0 \sin \omega t$  is applied across pure inductor of inductance  $L$ . The magnitude of induced emf is given by

$$e = L \frac{dI}{dt} \quad (1)$$

For the circuit,

Magnitude of induced emf = Applied voltage

i.e.  $L \frac{dI}{dt} = V_0 \sin \omega t$

or  $dI = \frac{V_0}{L} \sin \omega t \, dt$

On integrating both sides, we get

$$I = \frac{V_0}{L} \int \sin \omega t \, dt = \frac{V_0}{L} \left( \frac{-\cos \omega t}{\omega} \right) \quad (1)$$

or  $I = -\frac{V_0}{\omega L} \cos \omega t = -\frac{V_0}{\omega L} \sin \left( \frac{\pi}{2} - \omega t \right)$

$$\therefore I = \frac{V_0}{X_L} \sin \left( \omega t - \frac{\pi}{2} \right) \quad \dots(i)$$

where,  $X_L = \omega L =$  inducting reactance

$$I = I_0 \sin \left( \omega t - \frac{\pi}{2} \right) \quad \dots(ii)$$

where,  $I_0 =$  peak value of AC

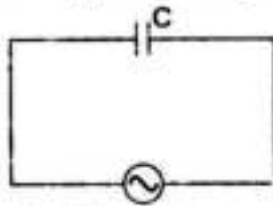
But,  $V = V_0 \sin \omega t \quad \dots(iii)$

From Eqs. (ii) and (iii), it is clear that AC lags behind the voltage by phase  $\frac{\pi}{2}$ . (1)

22. An AC voltage,  $V = V_0 \sin \omega t$  is applied across a pure capacitor,  $C$ . Obtain an expression for the current  $I$  in the circuit and hence obtain the
- capacitive reactance of the circuit and
  - the phase of the current flowing with respect to the applied voltage.
- [All India 2010C]

Ans.

- (i) Let alternating voltage,  $V = V_0 \sin \omega t$  is applied across a capacitor  $C$ . At any instant, the potential difference across the capacitor is equal to applied voltage. (1)



$$V = V_0 \sin \omega t \quad \dots(i)$$

$\therefore V =$  Potential difference across the capacitor  $= \frac{q}{C}$

$$\Rightarrow q = CV$$

$$\text{or } q = CV_0 \sin \omega t$$

$$\therefore \frac{dq}{dt} = \omega C V_0 \cos \omega t$$

$$\text{or } I = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \cos \omega t$$

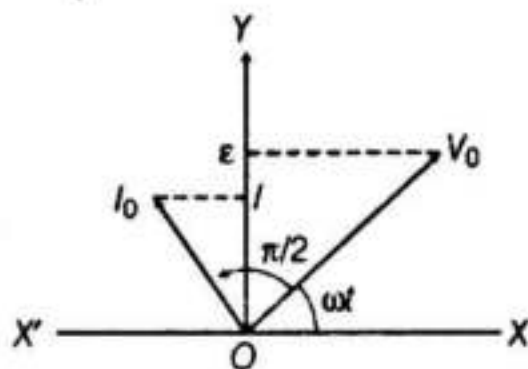
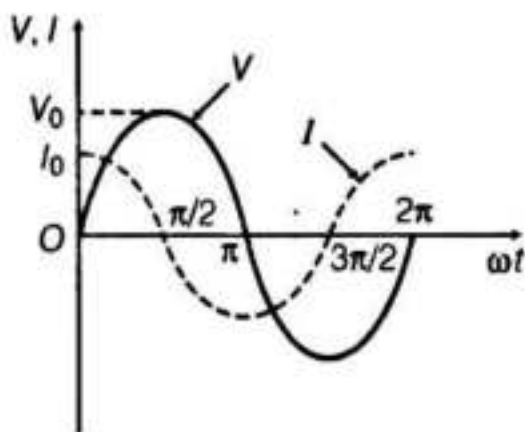
$$\text{or } I = I_0 \sin \left( \omega t + \frac{\pi}{2} \right) \quad \dots(ii)$$

$$\text{where, } I_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)} = \frac{V_0}{X_C}$$

where,  $I_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)} = \frac{V_0}{X_C}$

Capacitive reactance,  $X_C = \frac{1}{\omega C}$  **(1)**

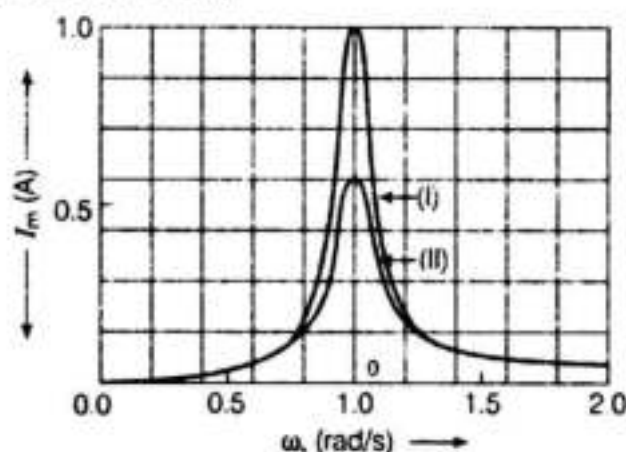
(ii) From Eqs. (i) and (ii), current leads the voltage by phase  $\frac{\pi}{2}$ .



**(1)**



23. The graphs shown here depict the variation of current  $I_{\text{rms}}$  with angular frequency  $\omega$  for two different series L-C-R circuits.



Observe the graphs carefully.

(i) State the relation between L and C values of the two circuits when the current in the two circuits is maximum.

(ii) Indicate the circuit for which

- power factor is higher
- quality factor Q is larger.

Give the reasons for each case. [Delhi 2009C]

Ans.

(i) Since, resonant angular frequency  $\omega_0$  is same for both the graphs.

$$\text{i.e. } (\omega_0)_1 = (\omega_0)_2 \quad \left( \because \omega_0 = \frac{1}{\sqrt{LC}} \right)$$

$$\Rightarrow \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$$

$$\Rightarrow L_1 C_1 = L_2 C_2$$

$$\text{or } \frac{L_1}{L_2} = \frac{C_2}{C_1} \quad (1)$$

(ii) (a) In graph (I), current  $I_{m_1}$  is greater than graph (II)  $I_{m_2}$ .

$$\text{i.e. } I_{m_1} > I_{m_2}$$

$$\Rightarrow R_1 < R_2$$

Power factor of circuit represented by graph (I) has power factor as  $\cos \phi \propto R$ .

(1)

$$(b) \because Q = \frac{1}{R} \sqrt{\frac{L}{C}} \quad [\text{For two graphs}]$$

$$\text{i.e. } Q \propto \frac{1}{R}$$

$$\text{As, } R_1 < R_2$$

$$\therefore Q_1 > Q_2$$

Quality factor of graph (I) is higher than that of graph (II). (1)

24. An inductor  $200 \text{ mH}$ , capacitor  $500 \mu\text{F}$ , resistor  $10 \Omega$  are connected in series with a  $100 \text{ V}$  variable frequency AC source. Calculate the
- (i) frequency at which the power factor of the circuit is unity.
  - (ii) current amplitude at this frequency.
  - (iii) Q-factor. [Delhi 2008; Foreign 2008]

Ans.

Given,  $L = 200 \text{ mH} = 2 \times 10^{-1} \text{ H}$

$$C = 500 \mu\text{F} = 5 \times 10^{-4} \text{ F}$$

- (i) When power factor  $\cos \phi = 1$

Then,  $\phi = 0$

$\Rightarrow$  L-C-R circuit is in resonance.

$$\begin{aligned}\therefore \text{Resonating frequency, } \omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{2 \times 10^{-1} \times 5 \times 10^{-4}}}\end{aligned}$$

$$\omega_0 = 100 \text{ s}^{-1} \quad (1)$$

- (ii) We know that,  $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R}$

[ $\because$  at resonance condition,  $Z = R$ ]

$$\therefore I_{\text{rms}} = \frac{100}{10} = 10 \text{ A}$$

$$\begin{aligned}\text{Amplitude of maximum current, } I_0 &= I_{\text{rms}} \sqrt{2} \\ &= 10 \sqrt{2} \text{ A} \quad (1)\end{aligned}$$

$$\begin{aligned}
 \text{(iii) As, } Q\text{-factor} &= \frac{1}{R} \sqrt{\frac{L}{C}} \\
 &= \frac{1}{10} \sqrt{\frac{2 \times 10^{-1}}{5 \times 10^{-4}}} = \frac{1}{10} \sqrt{0.4 \times 10^3} \\
 &= \frac{20}{10} = 2
 \end{aligned}
 \tag{1}$$

$\therefore$  Q-factor = 2

**25.** An inductor and a bulb are connected in series to an AC source of 12 V, 50 Hz. 2 A current flows in the circuit and the phase angle between voltage and current is  $\frac{\pi}{4}$  rad. Calculate the impedance and inductance of the circuit. [Foreign 2008]

Ans.

💡 To find out the inductance firstly, we have to calculate (Z) impedance and resistance. Here, resistance can be calculated by the given value of  $\phi$  and calculated value of Z means formula of power factor can be applied.

Given,  $V_{\text{rms}} = 12 \text{ V}$ ,  $f = 50 \text{ Hz}$ ,  $\phi = \frac{\pi}{4} \text{ rad}$

$$I_{\text{rms}} = 2 \text{ A}$$

$$\therefore \text{ Impedance, } Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{12 \text{ V}}{2 \text{ A}} = 6 \Omega \tag{1}$$

$$\text{Also, power factor, } \cos \phi = \frac{R}{Z}$$

$$\begin{aligned}
 \Rightarrow R &= Z \cos \phi \\
 &= 6 \times \cos \frac{\pi}{4} = 6 \times \frac{1}{\sqrt{2}} \\
 R &= 3\sqrt{2} \Omega
 \end{aligned}
 \tag{1/2}$$

In C-R circuit,  $R^2 + X_L^2 = Z^2$

$$X_L^2 = Z^2 - R^2 = (6)^2 - (3\sqrt{2})^2$$

$$\Rightarrow X_L = 3\sqrt{2} \Omega \tag{1/2}$$

$$\text{But, } X_L = 2\pi fL = 3\sqrt{2}$$

$$\begin{aligned}
 \text{Inductance, } L &= \frac{3\sqrt{2}}{2\pi f} = \frac{3\sqrt{2}}{2 \times 3.14 \times 50} \\
 &= 13.5 \text{ mH}
 \end{aligned}
 \tag{1}$$

26. A coil of  $L = 0.5\pi \Omega$  is connected to a 200 V,  $50 \text{ s}^{-1}$  AC source. Calculate the following
- Maximum current in the coil.
  - Time lag between voltage maximum and current maximum. [Foreign 2008]

Ans.

(i) Given,  $V_{\text{rms}} = 200 \text{ V}$ ,

$$f = 50 \text{ s}^{-1}, L = 0.5\pi \Omega$$

Inductive reactance,  $X_L = \omega L = 2\pi fL$

$$= 2\pi \times 0.5\pi \times 50 = 492.98 \Omega$$

$$= 493 \Omega$$

Maximum current,

$$I_0 = \frac{V_0}{X_L} = \frac{V_{\text{rms}}\sqrt{2}}{X_L} = \frac{200\sqrt{2}}{493}$$

$$= 0.4 \times 1.414 = 0.57 \text{ A} \quad (1)$$

(ii) Phase difference  $= \frac{2\pi}{T} \times \text{Time difference}$

$\therefore$  For pure inductor, phase difference

$$\phi = \frac{\pi}{2}, \quad T = \frac{1}{f} = \frac{1}{50} \text{ s} \quad (1)$$

$\therefore$  Time difference  $= \frac{T}{2\pi} \times \text{Phase difference}$

$$= \left( \frac{1}{50} \right) \times \frac{\pi}{2}$$

i.e. Time lag between  $V$  and  $I$  is  $\frac{1}{200} \text{ s}$  (1)

27. A coil of inductance, 0.5 H and resistance,  $100 \Omega$  is connected to a 200 V,  $50 \text{ s}^{-1}$  AC source. Calculate

- the maximum current in the coil.
- time lag between voltage maximum and current maximum. [Foreign 2008]

Ans. Refer Q.no 26

28. When a given coil is connected to a 200 V DC source, 2 A current flows and when the same coil is connected to 200 V, 50 Hz AC source, only 1 A current flows in the circuit.

- Explain why the current decreases in the latter case?
- Calculate the self-inductance of the coil used.

[Foreign 2008]

Ans.

💡 In DC circuit, an inductor offers no resistance, so the impedance of the circuit is equal to the resistance in the circuit ( $Z = R$ ).

(i) In DC circuit,  $X_L = 2\pi fL = 0$  ( $\because f = 0$ )

$\therefore$  But a coil have impedance,

$$Z = \sqrt{R^2 + X_L^2}$$

In DC circuit,  $Z = R$  (as  $X_L = 0$ )

But in AC,  $Z = \sqrt{R^2 + X_L^2} > R$

( $X_L$  have certain value in AC)

i.e.  $I \propto \frac{1}{Z}$

So, in DC more current flows than AC.

$\left(1\frac{1}{2}\right)$

(ii) In DC circuit,

$$R = \frac{V}{I} = \frac{200}{2} = 100 \Omega$$

$$\text{In AC circuit, } Z = \frac{V}{I} = \frac{200}{1} = 200 \Omega \quad (1)$$

$$\Rightarrow R^2 + X_L^2 = Z^2$$

$$X_L^2 = Z^2 - R^2 = (200)^2 - (100)^2$$

$$X_L = 100\sqrt{3} \Omega$$

$$\Rightarrow 2\pi fL = 100\sqrt{3} \Omega$$

$$\therefore \text{Inductance, } L = \frac{100\sqrt{3}}{2\pi f}$$

$$\therefore \text{Inductance, } L = \frac{100\sqrt{3}}{2\pi f}$$

$$\begin{aligned} \text{Given, } f &= 50\text{ Hz} \\ &= \frac{100\sqrt{3}}{2 \times 3.14 \times 50} \end{aligned}$$

$$L = 0.55 \text{ H} \quad (1/2)$$

**29.** A  $100 \mu\text{F}$  capacitor is in series with a  $40 \Omega$  resistor and is connected to a  $100 \text{ V}$ ,  $50 \text{ Hz}$  AC source. Calculate the following

- (i) Maximum current in the circuit.
- (ii) Time lag between current maximum and voltage maximum. [Foreign 2008]

**Ans.**

- 💡 (i) To calculate the maximum current in circuit, firstly find the impedance ( $Z$ ) and rms value of current.
- (ii) It can be calculated with the help of formula of phase difference.

(i) Given,  $V_{\text{rms}} = 100 \text{ V}$ ,  $C = 100 \mu\text{F}$ ,

$$C = 100 \times 10^{-6} \text{ F}$$

$$R = 40 \Omega \quad (1)$$

Total impedance of the circuit,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

$$= \sqrt{(40)^2 + \left(\frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}}\right)^2}$$

$$= 51.12 \, \Omega \quad (1)$$

$$\begin{aligned} \text{Current in the circuit, } I_{\text{rms}} &= \frac{V_{\text{rms}}}{Z} \\ &= \frac{100}{51.12} \\ &= 1.95 \, \text{A} \end{aligned}$$

$$\begin{aligned} \text{Maximum current, } I_0 &= I_{\text{rms}} \sqrt{2} \\ &= 1.95 \sqrt{2} \\ &= 2.76 \, \text{A} \end{aligned}$$

(ii) For pure capacitor, phase difference,  $\phi = \frac{\pi}{2}$

$$\begin{aligned} \text{Time difference} &= \frac{1}{2\pi} \times \text{Phase difference} \\ &= \frac{1}{2\pi} \times \frac{\pi}{2} = \frac{1}{200} \, \text{s} \quad (1) \end{aligned}$$

$\therefore$  Time lag between current maximum and voltage maximum =  $\frac{1}{200} \, \text{s}$ .

**30.** An AC source of voltage  $V = V_m \sin \omega t$  is connected one by one to three circuit elements X, Y and Z. It is observed that the current flowing in them

- (i) is in phase with applied voltage for element X.
- (ii) leads the voltage in phase by  $\pi/2$  for element Y.
- (iii) lags the applied voltage in phase by  $\pi/2$  for element Z.

Identify the three circuit elements.

Find an expression for the

- (a) current flowing in the circuit.
- (b) net impedance of the circuit when the same AC source is connected across a series combination of the elements, X, Y and Z.

If the frequency of the applied voltage is varied, then set up the condition of the frequency when the current amplitude in the circuit is maximum. Write the expression for this current amplitude. [HOTS; Delhi 2008C]

Ans.



We will identify the element according to the phase lead and lag with voltage or current.

- (i) In element X, current is flowing in phase with applied voltage, so element X is a resistance.
- (ii) In element Y, current leads the applied voltage in phase by  $\frac{\pi}{2}$ , so element Y is a capacitor.
- (iii) In element Z, current lags the applied voltage in phase by  $\frac{\pi}{2}$ , so element Z is an inductor. (1)

(a) Current flowing in the circuit,

$$I = I_0 \sin(\omega t + \phi), \text{ if } X_C > X_L$$

$$I = I_0 \sin(\omega t - \phi), \text{ if } X_L > X_C$$

$$\text{where, } \tan \phi = \frac{X_L - X_C}{R} \quad (1/2)$$

(b) Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{where, } X_L = \omega L, \quad X_C = \frac{1}{\omega C} \quad (1/2)$$

Current amplitude will be at resonance.

At resonance,  $X_L = X_C$

$$\therefore Z = R$$

Thus, maximum current of amplitude,

$$I_0 = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} \quad (1)$$

31. What does the term 'phasors' in AC circuit analysis mean? An AC source of voltage,  $V = V_m \sin \omega t$  is applied across a pure inductor of inductance L. Obtain an expression for the current I flowing in the circuit. Also draw the

- phasor diagram.
- graphs of V and I versus  $\omega t$  for this circuit. [Delhi 2008C]

Ans.



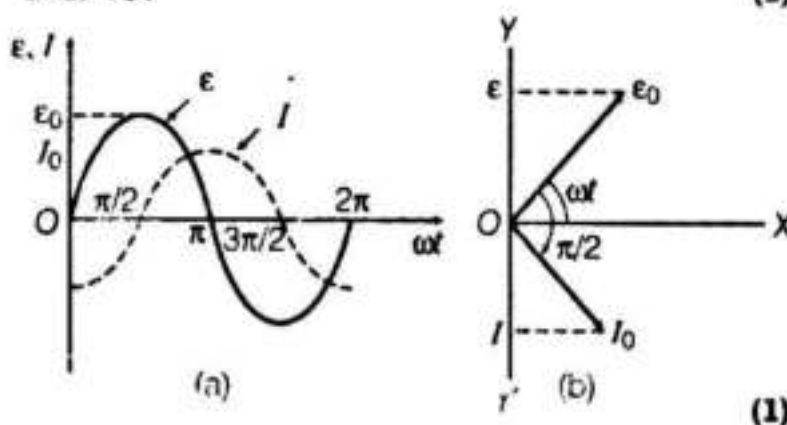
31. What does the term 'phasors' in AC circuit analysis mean? An AC source of voltage,  $V = V_m \sin \omega t$  is applied across a pure inductor of inductance  $L$ . Obtain an expression for the current  $I$  flowing in the circuit. Also draw the

- phasor diagram.
- graphs of  $V$  and  $I$  versus  $\omega t$  for this circuit. [Delhi 2008C]

Ans.

**Phasor** A phasor is a vector which rotates about the origin with an angular speed  $\omega$  in anti-clockwise direction that representing it as a sinusoidally varying quantity (like alternating emf and alternating current). Its length represents the amplitude of the quantity and projection on a fixed axis given the instantaneous value of represented quantity. (1)

**For explanation of the current** Refer to ans. 19. (1)



**32.** A series  $L$ - $C$ - $R$  circuit is made by taking  $R = 100\ \Omega$ ,  $L = \frac{2}{\pi}\text{ H}$ ,  $C = \frac{100}{\pi}\ \mu\text{F}$ . This series combination is connected across an AC source of 220 V, 50 Hz. Calculate

- (i) the impedance of the circuit and
- (ii) the peak value of the current flowing in the circuit.
- (iii) the power factor of this circuit and compare this value with the one at its resonant frequency. [All India 2008C]

Ans.



Firstly, to find out the impedance, we have to find Inductive reactance ( $X_L$ ) and capacitive reactance ( $X_C$ ), then can find peak value of current and power factor.

(i) Given,  $R = 100\ \Omega$ ,  $L = \frac{2}{\pi}\text{ H}$ ,  $C = \frac{100}{\pi}\ \mu\text{H}$ ,

$V = 200\text{ V}$ ,  $f = 50\text{ Hz}$ .

$\therefore$  Inductive reactance of AC circuit,

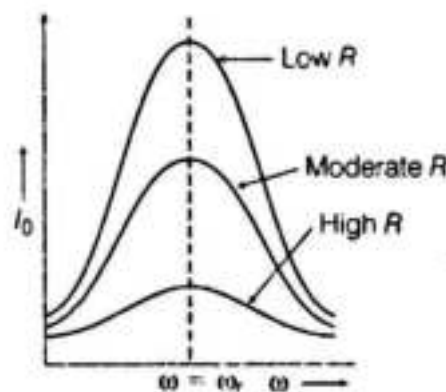
$$X_L = \omega L = 2\pi fL = 2\pi \times 50 \times \frac{2}{\pi} = 200\ \Omega$$

Capacitive reactance of AC circuit,

$$\begin{aligned} X_C &= \frac{1}{\omega C} = \frac{1}{2\pi fC} \\ &= \frac{1}{2\pi \times 50 \times \left(\frac{100}{\pi}\right) \times 10^{-6}} = 100\ \Omega \end{aligned}$$

💡 On increasing the frequency of AC, there will be no effect on resistance (as it does not depend on frequencies). Inductive reactance will increase (as  $X_L = 2\pi fL$  or  $X_L \propto f$ ), capacitive reactance will decrease (as  $X_C = \frac{1}{2\pi fC}$  or  $X_C \propto \frac{1}{f}$ ). If resistance or reactance of a circuit increases/decreases, the current for the circuit decreases/increases.

(i)



i.e. Quality factor,  $Q = \frac{\text{voltage across } L \text{ or } C}{\text{applied voltage}}$

$$Q = \frac{(\omega_r L)I}{RI}$$

[ $\because$  applied voltage = voltage across  $R$ ]

or  $Q = \frac{\omega_r L}{R}$

or  $Q = \frac{(1/\omega_r C)I}{RI} = \frac{1}{RC\omega_r}$

$\therefore Q = \frac{L}{RC \cdot \frac{1}{\sqrt{LC}}} \text{ [using } \omega_r = \frac{1}{\sqrt{LC}} \text{]}$

$$= \frac{1}{R} \sqrt{\frac{L}{C}}$$

or  $Q = \frac{\sqrt{LC}}{RC} = \frac{1}{R} \sqrt{\frac{L}{C}} \text{ [using } \omega_r = \frac{1}{\sqrt{LC}} \text{]}$

Thus,  $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$

From the phasor diagram,

$$V = V_L + V_R + V_C$$

Magnitude of net voltage,

$$V_m = \sqrt{(V_{rm})^2 + (V_{cm} - V_{lm})^2}$$

$$V_m = I_m \sqrt{R^2 + (X_C - X_L)^2}$$

From the figure,

$$\tan \phi = \frac{V_{cm} - V_{lm}}{V_{rm}}$$

(2)

$$= \frac{I_m (X_C - X_L)}{I_{mr}}$$

$$\phi = \tan^{-1} \left( \frac{X_C - X_L}{R} \right)$$

(b) Refer to ans. 19.

(3)

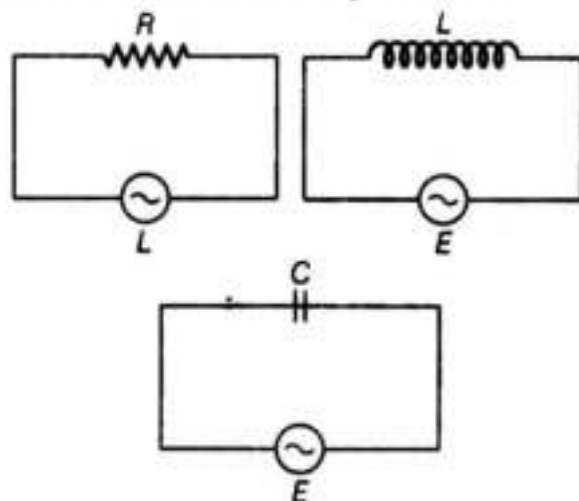
34. Derive an expression for the impedance of a series L-C-R circuit connected to an AC supply of variable frequency.

Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set? [Delhi 2011]

Ans. Refer to ans. 19 and for resonance condition refer to ans. 30(b)

35. (i) What do you understand by sharpness of resonance in a series L-C-R circuit? Derive an expression for Q-factor of the circuit.

(ii) Three electrical circuits having AC sources of variable frequency are shown in the figures. Initially, the current flowing in each of these is same. If the frequency of the applied AC source is increased, how will the current flowing in these circuits be affected? Give the reason for your answer.



[HOTS; Delhi 2011C]

Ans.

On increasing the frequency of AC, there will be no effect on resistance (as it does not depend on frequencies). Inductive reactance will increase (as  $X_L = 2\pi fL$  or  $X_L \propto f$ ), capacitive reactance will decrease (as  $X_C = \frac{1}{2\pi fC}$  or  $X_C \propto \frac{1}{f}$ ). If resistance or reactance of a circuit

Impedance of L-C-R series AC circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(100)^2 + (200 - 100)^2}$$

$$= 100\sqrt{2} \, \Omega \quad (1)$$

(ii) Peak value of current flowing through the circuit,  $I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{100\sqrt{2}}$

$$(\because V_{rms} = 200 \, V, V_0 = \sqrt{2}V_{rms} = 200\sqrt{2} \, V)$$

$$I_0 = 2.2 \, A \quad (1)$$

(iii) Power factor,  $\cos \phi = \frac{R}{Z}$

$$\cos \phi = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore$  Power factor at resonance,  $\cos \phi = 1$

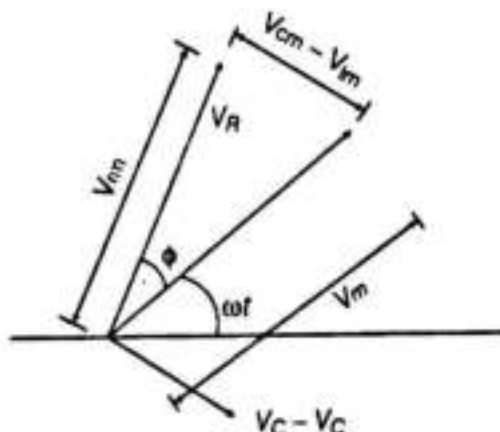
$$\text{For comparison, } \frac{(\cos \phi)_{\text{given circuit}}}{(\cos \phi)_{\text{resonant circuit}}} = \frac{1}{\sqrt{2}} \quad (1)$$

## 5 Marks Questions

33. (i) A series L-C-R circuit is connected to an AC source of variable frequency. Draw a suitable phasor diagram to deduce the expressions for the amplitude of the current and phase angle.

(ii) Obtain the condition at resonance. Draw a plot showing the variation of current with the frequency of AC source for two resistances  $R_1$  and  $R_2$  ( $R_1 > R_2$ ). Hence, define the quality factor  $Q$  and write its role in the tuning of the circuit. [Delhi 2014C]

Ans.



(iii) Impedance becomes minimum and equal to ohmic resistance

$$\text{i.e. } Z = Z_{\text{minimum}} = R$$

(iv) AC becomes maximum,

$$\therefore I_{\text{max}} = \frac{V_{\text{max}}}{Z_{\text{min}}} = \frac{V_{\text{max}}}{R} \quad (1)$$

(v) Voltage and current arrives in same phase.

**Power factor** In an AC circuit, the ratio of true power consumption and virtual power consumption is termed as power factor.

$$\begin{aligned} \text{i.e. } \cos \phi &= \frac{P_{\text{av}}}{V_{\text{rms}} I_{\text{rms}}} \\ &= \frac{\text{True power}}{\text{Apparent power}} \end{aligned}$$

$$\begin{aligned} \text{Also, } \cos \phi &= \frac{R}{Z} \\ &= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (1) \end{aligned}$$

The power factor is maximum.

i.e.  $\cos \phi = +1$ , in L-C-R series AC circuit when circuit is in resonance. The power factor is minimum when phase angle between  $V$  and  $I$  is  $90^\circ$ , i.e. either pure inductive circuit or pure capacitive AC circuit. (1)

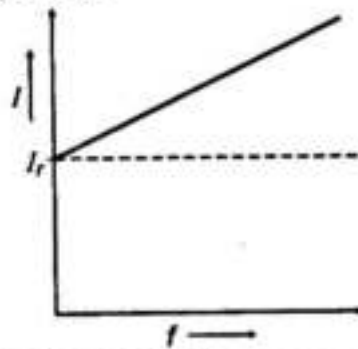
**37.** A resistor of  $400 \, \Omega$ , an inductor of  $\frac{5}{\pi} \, \text{H}$

and a capacitor of  $\frac{50}{\pi} \, \mu\text{F}$  are connected

in series across a source of alternating voltage of  $140 \sin 100 \pi t \, \text{V}$ . Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

For given circuit,  $I \propto f$

Current increases with the increase of frequency.



(1)

36. A series L-C-R circuit is connected to an AC source having voltage  $V = V_m \sin \omega t$ . Derive the expression for the instantaneous current  $I$  and its phase relationship to the applied voltage. Obtain the condition for resonance to occur. Define power factor. State the conditions under which it is

- maximum and
- minimum. [All India 2010]

Ans.

Phase difference between voltage and current,

$$\tan \phi = \frac{X_L - X_C}{R} \quad \dots(i)$$

and 
$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$$

$\therefore$  Expression of AC,

$$I = I_0 \sin(\omega t - \phi) \quad (1)$$

**Conditions for resonance**

(i) Inductive reactance must be equal to capacitive reactance

i.e.  $X_L = X_C$

(ii) As,  $X_L = X_C$

$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$$

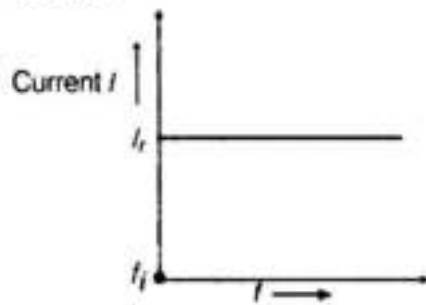
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

where,  $\omega_0$  = resonant angular frequency.

(1)

(iii) Impedance becomes minimum and equal to ohmic resistance

- (a) **Circuit containing resistance  $R$  only** There will not be any effect in the current on changing the frequency of AC source. (1)



where,  $f_i$  = initial frequency of AC source.

There is no effect on current with the increase in frequency.

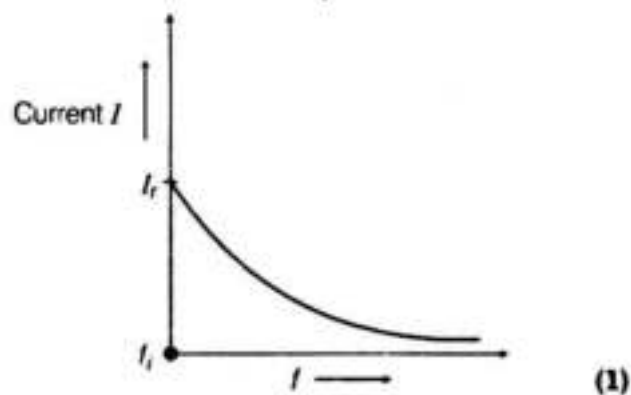
- (b) **AC circuit containing inductance only** With the increase of frequency of AC source inductive reactance increase as

$$I = \frac{V_{rms}}{X_L}$$

$$= \frac{V_{rms}}{2\pi fL}$$

For given circuit,

$$I \propto \frac{1}{f}$$



Current decreases with the increase of frequency.

- (c) **AC circuit containing capacitor only**

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

$$\text{Current, } I = \frac{V_{rms}}{X_C} = \left( \frac{V_{rms}}{\frac{1}{2\pi fC}} \right)$$

$$I = 2\pi fC V_{rms}$$

For given circuit,  $I \propto f$

Current increases with the increase of



$$\phi = \tan^{-1} \left( \frac{\text{Reactive impedance}}{\text{Resistance}} \right)$$

**[All India 2009]**

Ans.

**For impedance** Refer to ans. 19. **(1)**

Also, from phasor diagram

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{\text{Reactive impedance}}{\text{Resistance}}$$

$$\Rightarrow \phi = \tan^{-1} \left[ \frac{\text{Reactive impedance}}{\text{Resistance}} \right] \quad \textbf{(1)}$$

The impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where,  $Z = R$

$$\Rightarrow R = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow (X_L - X_C)^2 = 0$$

$$X_L = X_C$$

$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \textbf{(3)}$$

where,  $\omega_0$  is resonant  $L$ - $C$ - $R$  series AC circuit.

$$V_{rms} \text{ across resistor } R, V_R = I_{rms} R \\ = 0.2 \times 400 = 80 \text{ V}$$

$$V_{rms} \text{ across inductor, } V_L = I_{rms} X_L \\ = 0.2 \times 500 = 100 \text{ V}$$

$$V_{rms} \text{ across capacitor, } V_C = I_{rms} \times X_C \\ = 0.2 \times 200 = 40 \text{ V} \quad (2)$$

$$\text{Here, } V = V_R + V_L + V_C$$

Because  $V_L$  and  $V_R$  are not in same phase,

$$\therefore V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

**38. (i)** Derive an expression for the average power consumed in a series  $L$ - $C$ - $R$  circuit connected to AC source for which the phase difference between the voltage and the current in the circuit is  $\phi$ .

**(ii)** Define the quality factor in an AC circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends. [Delhi 2009]

**Ans. (i)** Refer to ans 12(i)

**(ii)** Refer to ans. 35 (i)

Quality factor depends on ohmic resistance of  $L$ - $C$ - $R$  Series Ac and Inductance and Capacitance of  $L$ - $C$ - $R$  Series AC Circuit.

**39.** An AC source of emf,  $E = E_0 \sin \omega t$  is connected across a series combination of an inductor  $L$ , a capacitor  $C$  and a resistor,  $R$ . Obtain an expression for the equivalent impedance  $Z$  of the circuit and hence, find the value of  $\omega$  for the AC source for which  $Z = R$ .

Show that the phase angle  $\phi$  (between the current flowing in this circuit and the voltage applied to it) can be obtained through the relation

and a capacitor of  $\frac{50}{\pi} \mu\text{F}$  are connected in series across a source of alternating voltage of  $140 \sin 100 \pi t$  V. Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

(Given,  $\sqrt{2} = 1.414$ ).

[Foreign 2010]

Ans.



To find the voltage across each circuit element, following steps to be followed are:

- (i) To calculate the maximum current in circuit, firstly find the (Z) impedance and rms value of current.
- (ii) It can be calculated with the help of formula of phase difference.

As, applied voltage

$$V = 140 \sin 100 \pi t$$

$$C = \frac{50}{\pi} \mu\text{F}, L = \frac{5}{\pi} \text{H}, R = 400 \Omega$$

$$= \frac{50}{4} \times 10^{-6} \text{F}$$

Comparing it with  $V = V_0 \sin \omega t$ ,

$$V_0 = 140 \text{ V}, \omega = 100 \pi$$

Inductive reactance,  $X_L = \omega L$

$$X_L = 100 \pi \times \frac{5}{\pi} = 500 \Omega$$

Capacitive reactance,  $X_C = \frac{1}{\omega C}$

$$X_C = \frac{1}{100 \pi \times \frac{50}{\pi} \times 10^{-6}} = 200 \Omega$$

Impedance of the AC circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(400)^2 + (500 - 200)^2}$$

$$Z = \sqrt{1600 + 900} = 500 \Omega \quad (2)$$

Maximum current in the circuit,

$$I_0 = \frac{V_0}{Z} = \frac{140}{500}$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{140}{500 \times \sqrt{2}} = 0.2 \text{ A}$$

(1)

∴

$$V = \sqrt{V_R^2 + V_C^2}$$

(1/2)

## AC Devices

### 1 Mark Questions

**1. Mention the two characteristic properties of the material suitable for making core of a transformer. [All India 2012]**

**Ans.** (i) Low retentivity or coercivity.

(ii) Low hysteresis loss or high permeability and susceptibility.

**2. What is the function of a step-up transformer? [All India 2011C]**

**Ans.** Step-up transformer converts low alternating voltage into high alternating voltage and high alternating current into low alternating current. The secondary coil of step-up transformer has a greater number of turns than the primary ( $N_s > N_p$ ).

**3. Write any two factors responsible for energy losses in actual transformers. [Delhi 2009c]**

**Ans.** Two factors for energy losses are

(i) eddy currents

(ii) hysteresis loss

### 2 Marks Questions

**4. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers? [All India 2012]**

**Ans. Principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

**Power transmission** Electric power is transmitted over long distances at high voltage. So, step-up transformers are used at power stations to increase the voltage of power whereas a series of step-down transformers are used to decrease the voltage up to 220 V.

- 40.** An AC source generating a voltage,
- (i)  $V = V_m \sin \omega t$  is connected to a capacitor of capacitance,  $C$ . Find the expression for the current  $I$ , flowing through it. Plot a graph of  $V$  and  $I$  versus  $\omega t$  to show that the current is  $\pi/2$  ahead to a voltage.
- (ii) A resistor of  $200 \Omega$  and a capacitor of  $15.0 \mu\text{F}$  are connected in series to a  $200\text{V}$ ,  $50\text{Hz}$  AC source. Calculate the current in the circuit and the rms voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox. [All India 2008]

Ans.

(i) Refer to ans. 22. (2)

(ii) Given,  $R = 200 \Omega$ ,  $C = 40 \mu\text{F}$ ,  $V_{\text{rms}} = 220 \text{ V}$ ,  
 $\omega = 300 \text{ Hz}$

Inductive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{300 \times 40 \times 10^{-6}} \quad (1/2)$$

$$= 83.33 \Omega$$

$$\text{Impedance, } Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(200)^2 + (83.33)^2}$$

$$= 216.6 \Omega \quad (\text{nearly})$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{216.6} = 1.02 \text{ A} \quad (1/2)$$

Rms voltage across  $R$ ,

$$V_R = IR = 1.02 \times 200 = 204 \text{ V} \quad (1)$$

Across capacitor,

$$V_C = I X_C = 1.02 \times 83.33$$

$$V_C = 85 \text{ V}$$

$$V \neq V_R + V_C$$

Because  $V_C$  and  $V_R$  are not in same phase

(1/2)

Hence,  $V_R$  leads  $V_C$  by phase  $\frac{\pi}{2}$ .

(iii) Main assumptions

- The primary resistance and current are small.
- The flux linked with primary and secondary coil is same where there is no leakage of flux from the core.
- secondary current is small

(iv) Energy losses in transformer:

(i) **Eddy current loss** Eddy current in iron core of transformer facilitates the loss of energy in the form of heat.

(ii) **Flux leakage** Total fluxes linked with primary do not completely pass through the secondary which denotes the loss in the flux.

(iii) **Copper loss** Due to heating, energy loss takes place in copper wires of primary and secondary coils.

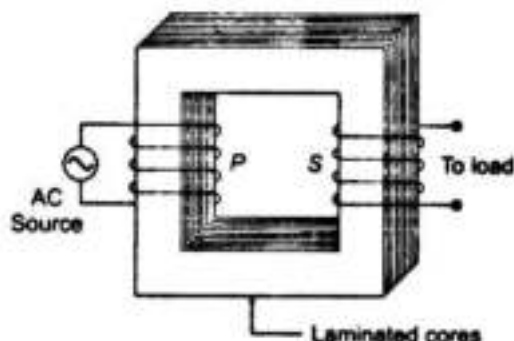
(iv) **Hysteresis loss** The energy loss takes place in magnetizing and demagnetizing the iron core over every cycle.

(v) **Humming loss** The magnetostriction effect leads to set core in vibration which in turn produces the sound. This loss is referred to as humming loss.

8. (i) State the principle of a step-up transformer. Explain with the help of a labelled diagram, its working.

(ii) Describe briefly two energy losses giving the reasons for their occurrence in actual transformer. [Foreign 2012]

**Ans. Principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.



9. With the help of a labelled diagram, describe briefly the underlying principle and working of a step-up transformer.

(ii) Write any two sources of energy loss in a transformer, (iii) A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy? Explain. [Delhi 2011]

## 5 Marks Questions

7. (i) Draw a schematic arrangement for winding of primary and secondary coils in a transformer when the two coils are wound on top of each other.

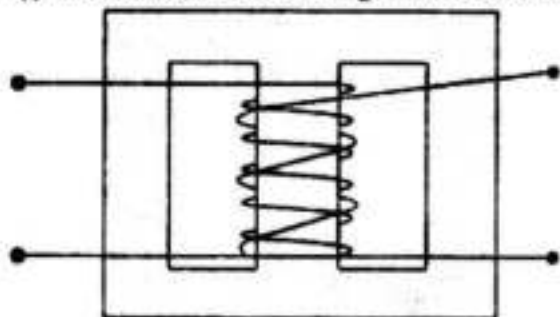
(ii) State the underlying principle of a transformer and obtain the expression for the ratio of secondary to primary voltage in terms of the

- number of secondary and primary windings and
- primary and secondary currents.

(iii) Write the main assumption involved in deriving the above relations.

(iv) Write any two reasons due to which energy losses may occur in actual transformers. [HOTS]

**Ans.** (i) The schematic arrangement of a transformer is shown as below:



(ii) Principle of transformer

Refer to ans. 4.

(2)

**Working** when an alternating current is passed through the primary, the magnetic flux through the iron core changes, which does two things, produces emf in the primary and an induced emf is set up in the secondary. If we assume that the resistance of primary is negligible, then the back emf will be equal to the voltage applied to the primary.

$$(a) \therefore V_1 = -N_1 \frac{d\phi}{dt}$$

$$\text{and } V_2 = -N_2 \frac{d\phi}{dt}$$

where,  $N_1$  and  $N_2$  are number of turns in the primary and the secondary coils respectively while  $V_1$  and  $V_2$  are their voltages respectively.

(b) But for ideal transformers,

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

## 2 Marks Questions

**4.State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers? [All India 2012]**

**Ans. Principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

**Power transmission** Electric power is transmitted over long distances at high voltage. So, step-up transformers are used at power stations to increase the voltage of power whereas a series of step-down transformers are used to decrease the voltage up to 220 V.

**5.State the principle of working of a transformer. Can a transformer be used to step-up or step-down a DC voltage? Justify your answer. [All India 2011]**

**Ans. (i) For principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

(ii) No, transformer cannot be used to change DC voltage. This happens because DC voltage cannot change flux linked with primary or secondary coils.

**6. Mention various energy losses in a transformer. [All India 2011]**

**Ans.** Energy losses in transformer:

(i) **Eddy current loss** Eddy current in iron core of transformer facilitates the loss of energy in the form of heat.

(ii) **Flux leakage** Total fluxes linked with primary do not completely pass through the secondary which denotes the loss in the flux.

(iii) **Copper loss** Due to heating, energy loss takes place in copper wires of primary and secondary Coils.

(iv) **Hysteresis loss** The energy loss takes place in magnetizing and demagnetising the iron core over every cycle.

(v) **Humming loss** The magnetostriction effect leads to set core in vibration which in turn produced the sound. This loss is referred as humming loss.



9. With the help of a labelled diagram, describe briefly the underlying principle and working of a step-up transformer.

(ii) Write any two sources of energy loss in a transformer, (iii) A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy? Explain. [Delhi 2011]

**Ans. (i) Principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

(ii) Two factors for energy losses are

- eddy currents
- hysteresis loss

(iii) No, it does not violate the law of conservation of energy because voltage increase is accompanied by decrease in it. In alternative, current in such a way for an ideal transformer input power equals to the output power.

10. (i) Draw a schematic diagram of a step-up transformer. Explain its working principle. Assuming the transformer to be 100% efficient, obtain the relation for

- the current in the secondary in terms of the current in the primary and
- the number of turns in the primary and secondary windings.

(ii) Mention two important energy losses in actual transformers and state how these can be minimized? [All India 2009C; Foreign 2009]

**Ans. Principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

(a) Let alternating current, alternating voltage and number of turns in primary and secondary coils are  $(I_p, I_s)$ ,  $(V_p, V_s)$  and  $(N_p, N_s)$ , respectively.

For 100% efficient transformer,

Input power = Output power

$$\Rightarrow V_p I_p = V_s I_s$$

Also, assuming transformation ratio of transformer is  $r$ ,

$$r = \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$\Rightarrow I_p = \frac{V_s}{V_p} \times I_s \quad (1/2)$$

$$(b) \quad r = \frac{N_s}{N_p} \quad (1/2)$$

(ii) Two energy losses in the actual transformers

(ii) Two energy losses in the actual transformers

- **Hysteresis loss** Loss of energy in magnetising and demagnetising the core of transformer in every cycle. It can be minimised by taking core of soft iron having low coercivity, retentivity and hysteresis loop.
- **Eddy current** Energy loss in the form of heat due to eddy current. It can be minimised by taking laminated core consisting of insulated rectangular sheets, piled-up one over another.

**11. Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils.**

**In an ideal transformer, how is this relation related to the currents in the two coils.**

**How is the transformer used in large scale transmission and distribution of electrical energy over the long distances? [All India 2010]**

**Ans.**

**Principle of transformer** A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

- (a) Let alternating current, alternating voltage and number of turns in primary and secondary coils are  $(I_p, I_s)$ ,  $(V_p, V_s)$  and  $(N_p, N_s)$ , respectively.

For 100% efficient transformer,

Input power = Output power

$$\Rightarrow V_p I_p = V_s I_s$$

Also, assuming transformation ratio of transformer is  $r$ ,

$$r = \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$\Rightarrow I_p = \frac{V_s}{V_p} \times I_s \quad (1/2)$$

$$(b) \quad r = \frac{N_s}{N_p} \quad (1/2)$$

(ii) Two energy losses in the actual transformers

- **Hysteresis loss** Loss of energy in magnetising and demagnetising the core of transformer in every cycle. It can be minimised by taking core of soft iron having low coercivity, retentivity and hysteresis loop.

- **Eddy current** Energy loss in the form of heat due to eddy current. It can be minimised by taking laminated core consisting of insulated rectangular sheets, piled-up one over another.

Step-up transformers are used at generating stations so as to transmit the power at high voltage to minimise the loss in the form of heat, whereas series of step-up transformers are used at receiving ends

Also, assuming transformation ratio of transformer is  $r$ .

$$r = \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$\Rightarrow I_p = \frac{V_s}{V_p} \times I_s \quad (1/2)$$

$$(b) \quad r = \frac{N_s}{N_p} \quad (1/2)$$

(ii) Two energy losses in the actual transformers

- **Hysterisis loss** Loss of energy in magnetising and demagnetising the core of transformer in every cycle. It can be minimised by taking core of soft iron having low

coercivity, retentivity and hysteresis loop.

- **Eddy current** Energy loss in the form of heat due to eddy current. It can be minimised by taking laminated core consisting of insulated rectangular sheets, piled-up one over another.

Step-up transformers are used at generating stations so as to transmit the power at high voltage to minimise the loss in the form of heat, whereas series of step-up transformers are used at receiving ends

**12 .A step-up down transformer operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 :1. If the transformer is 90% efficient, calculate**

- the power output
- the voltage and
- the current in the secondary coil. [Foreign 2010]

Given, input voltages,  $V_p = 2.5 \times 10^3 \text{ V}$  (3)

Input current,  $I_p = 20 \text{ Asss}$

$$\text{Also, } \frac{N_p}{N_s} = \frac{10}{1} \Rightarrow \frac{N_s}{N_p} = \frac{1}{10} \quad \dots(i) \quad (1/2)$$

Percentage efficiency

$$\begin{aligned} &= \frac{\text{Output power}}{\text{Input power}} \times 100 \\ \Rightarrow \quad \frac{90}{100} &= \frac{\text{Output power}}{V_p I_p} \end{aligned}$$

$$\begin{aligned} (i) \quad \text{Output power} &= \frac{90}{100} \times (V_p I_p) \\ &= \frac{90}{100} \times (2.5 \times 10^3) \times (20 \text{ A}) = 4.5 \times 10^4 \text{ W} \quad (1/2) \end{aligned}$$

$$(ii) \quad \therefore \frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = \frac{N_s}{N_p} \times V_p$$

$$\text{Voltage } V_s = \frac{1}{10} \times 2.5 \times 10^3 \text{ V} = 250 \text{ V} \quad (1/2)$$

$$(iii) \quad V_s I_s = 4.5 \times 10^4 \text{ W}$$

$$\text{Current, } I_s = \frac{4.5 \times 10^4}{V_s} = \frac{4.5 \times 10^4}{250}$$

$$I_s = 180 \text{ A} \quad (1/2)$$