Solution

Section A

1. Correct option – c: Both force and torque
   An electric dipole placed in a non-uniform electric field will experience both force and torque.

2. Correct option – b: 2/3
   $\Phi = q/\varepsilon$
   $= (2q/\varepsilon_0)/(3q/\varepsilon_0)$
   Thus, $N_1/N_2 = 2/3$

3. Correct option – b: $I_R = I_G$
   From given diagram, we can observe that when switch is closed or open the reading on galvanometer is same and resistance P and R are not same.
   Using the balanced condition, we can say that
   Current in resistance R and G will be same.
   Thus, $I_R = I_G$

4. Correct option – d:
   Given that,
   Length of wire 1, $L_1 = L$
   Diameter of 1st wire, $D_1 = 2D$
   Area of cross-section, $A_1 = \pi\left(\frac{2D}{2}\right)^2 = \pi D^2$
   $\therefore$ Resistance of wire 1, $R_1 = \frac{\rho L}{A} = \frac{\rho L}{\pi D^2}$
   Similarly,
   Length of wire 2, $L_2 = 2L$
   Diameter of 2nd wire, $D_2 = 3D$
   Area of cross-section, $A_2 = \pi\left(\frac{3D}{2}\right)^2 = \frac{9}{4} \times \pi D^2 = \frac{9}{4}A$
   $\therefore R_2 = \frac{\rho(2L)}{\frac{9}{4}A} = \frac{8}{9}R$
   Hence, the ratio of the potential difference of two wires are
   $\frac{V_1}{V_2} = \frac{IR}{\frac{8}{9}IR} = \frac{9}{8}$

5. Correct option – a: $\frac{5}{7}$
   Given that,
   $N_1 = 30$
   $A_1 = 3.6 \times 10^{-3}\ m^2$
   $B_1 = 0.25\ T$
   $N_2 = 42$
   $A_2 = 1.8 \times 10^{-3}\ m^2$

$B_2 = 0.5$ T

Now,

$$\frac{I_2}{I_1} = \frac{N_2B_2A_2}{N_1B_1A_1} = \frac{42 \times 0.5 \times 1.8 \times 10^{-3}}{30 \times 0.25 \times 3.6 \times 10^{-3}} = \frac{7}{5}$$

$$\therefore \frac{I_1}{I_2} = \frac{5}{7}$$

6. Correct option – d: $\frac{7}{16}$, into the plane of the paper.

   Now for the given figure, the magnetic field at centre M due to current flowing through DA will be

   $$B_{DA} = \frac{\mu_0 I}{4\pi R} \times \left(\frac{3\pi}{2}\right) = \frac{3\mu_0 I}{8R}$$

   Similarly, the magnetic field due to the current flowing through BC is

   $$B_{BC} = \frac{\mu_0 I}{4\pi(2R)} \times \left(\frac{\pi}{2}\right) = \frac{\mu_0 I}{16R}$$

   Whereas, magnetic field due to AB and CD will be zero since point M lies on the straight line.

   Hence net magnetic field for the given case will be

   $B = B_{DA} + B_{BC} + B_{AB} + B_{CD} = \left(\frac{3\mu_0 I}{8R} + \frac{\mu_0 I}{16R} + 0 + 0\right) = \frac{7\mu_0 I}{16R}$ And by using the right-hand thumb rule we can conclude that direction of the magnetic field will be in the plane of the paper.

7. Correct option – d:

   a dielectric is introduced into the gap between the plates of the capacitor.

   From the given condition, the effective resistance for two circuits is

   $$R_1 = \sqrt{r^2 + \left(\frac{1}{\omega C}\right)^2} \; \& \; R_2 = \sqrt{r^2 + (\omega L)^2}$$

   Based on the preceding equations, we may conclude that increasing the gap between the plates of the capacitor reduces capacitance, which causes R1 to increase, resulting in a decrease in current.

   As a result, the brightness of the bulb will diminish because its brightness is directly proportional to the current running through it.

   Hence, the bulb will glow brighter when a dielectric is introduced into the gap between the plates of the capacitor.

8. Correct option – d: 250 V

   Given that,

   f = 50 Hz

   L = 318 mH = 0.318

   R = 75 Ω

   V_R = 150 V

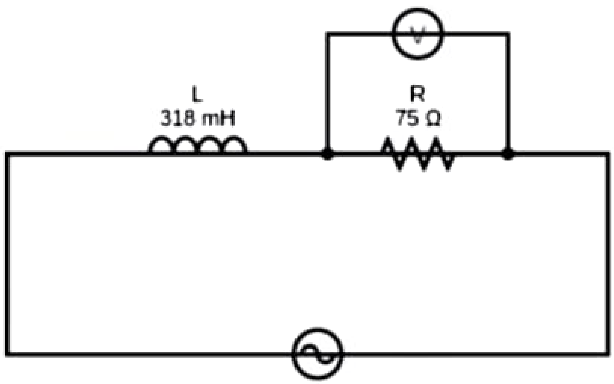
   Current through R, $I = \frac{V}{R} = \frac{150}{75} = 2$ A

   Now,

   $X_L = 2\pi fL = 2\pi \times 50 \times 0.318 \approx 100\ \Omega$

   $V_L = IX_L = 2 \times 100 = 200$ V

   Hence the total voltage, $V_{total} = (V_R^2 + V_L^2)^{\frac{1}{2}} = \sqrt{150^2 + 200^2} = 250$ V



9. Correct option – d: $\frac{1}{r^2}$

Now,

Electric potential for a dipole is given as

$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \Rightarrow V \propto \frac{1}{r^2}$

10. Correct option – b: k

The force between two small spheres is given as

$F_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ Whereas the force between them in the dielectric medium is

$F_2 = \frac{q_1 q_2}{4\pi k\epsilon_0 r^2} \ldots (\because \epsilon = k\epsilon_0)$

$\therefore \frac{F_1}{F_2} = \frac{1}{1/k} = k$

11. Correct option – d: a gap only

Taking out the infinity plug introduces the air gap in the circuit. Since air is a bad conductor of electricity, no current flows, implying that the infinity resistance in the circuit is introduced.

12. Correct option – b: 270 J

Given that,

Voltage, V = 15 V

Time, t =60 s

Internal resistance, r = 50 Ω

Now,

Heat energy dissipated in time t is given gas

E =VIt

i.e., $E = \frac{V^2}{r}t \ldots (\because V = Ir)$

$E = \frac{15^2}{50} \times 60 = 270\ J$

13. Correct option – a: Energy

Lenz's law is based on the principle of conservation of energy.

14. Correct option – b: 30°

Given that,

$B_v = \frac{1}{\sqrt{3}} B_H$

Also, $\tan\theta = \frac{B_V}{B_H} = \frac{\frac{B_H}{\sqrt{3}}}{B_H} = \frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

15. Correct option – a: 1.2 μT, Vertically upward

Given that,

Current, I = 15 A

Distance, d = 2.5 m

Now,

The magnitude of the magnetic field, $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 15}{2\pi \times 2.5}$

$\therefore B = 1.2\ \mu T$

And according to the right-hand thumb rule, the direction magnetic field will be upward.

16. Correct option – c: 10^2 V

Given that,

Inductance, L = 10 H

ΔI = 11 - 2 = 9 A

Time, t = 0.9 sec

Now,

$\epsilon = -L\frac{dI}{dt} = -\frac{10 \times 9}{0.9} = -100\ V$

17. Correct option – d: $-\frac{q}{4}$

A charge Q is placed at the centre of the line joining two charges q and q as shown below.

Now, for the system to be in equilibrium force between charges must be zero (ie., F =0).

$\therefore \frac{kq^2}{4x^2} + \frac{kqQ}{x^2} = 0 \Rightarrow Q = -\frac{q}{4}$

18. Correct option – c: $\vec{E}.d\vec{A}$

For the given case we know that electric flux is the dot product of the electric field and area of the cross section.

i.e., $\vec{\phi} = \vec{E}.d\vec{A}$

19. Correct option – d: 2 Ω

Given that,

Balancing length, l_1 = 120 cm

Shunted resistance, R = 1 Ω

New balancing length after shunting, l_2= 40 cm

Now,

Internal resistance, $r = R\left(\frac{120-40}{40}\right) = 2\ \Omega$

20. Correct option – d:

The electron will continue to move with the same velocity v along the axis of the solenoid.

As we know when an electron is projected with velocity v along the axis of a current-carrying long solenoid, it will continue to move with the same velocity v along the axis of the solenoid.

21. Correct option – d: Become half

Now,

$r = \frac{mv}{qB} \Rightarrow v \propto r$

22. Correct option –a:

It is placed in a space varying magnetic field that does not vary with time.

A metal plate can be heated by any of the flowing methods.

- Passing DC or AC through plates
- Keeping it time-varying magnetic field which produces induced current.

23. Correct option – d: Zero

Now,

Power = VI or EI

$\therefore P = E_0 I_0 \sin\omega t \times \sin\left(\omega t + \frac{\pi}{2}\right) = \text{zero}$

24. Correct option – a: $6 \times 10^6 m/s$

Given that,

Potential difference, V =100 V

Charge of electron, e = 1.6×10^{-19} C

Now,

$eV = \frac{1}{2} m_e v^2 \Rightarrow v = \sqrt{\frac{2eV}{m_e}} = 6 \times 10^6 m/s$

25. Correct option – c: A stationary charge

As we know charges at rest or rest remain unaffected by magnetic fields since force depends on velocity, and velocity is zero in the case of stationary charges.

Section B

26. Correct option – d: 2L

Given that,

Two-point charge +16q and -4q and located at x =0 and x = L

Now, we know that

$E = \frac{Q}{4\pi\epsilon_0 r^2} \Rightarrow \frac{16}{(L+x)^2} = \frac{-4}{x^2}$

$\therefore x = 2L$

27. Correct option – a:

Given that,

Dipole moment, $p = 4\times 10^{-5}$ C-m.

Electric field, $E = 10^{-3}$ N/C

Torque, $\tau = 2\times 10^{-8}$ Nm

Now,

The torque due to dipole is given as

$\tau = pE\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{\tau}{pE}\right)$

$\therefore \theta = \sin^{-1}\left(\frac{2\times10^{-8}}{4\times10^{-5}\times10^{-3}}\right) = 30°$

28. Correct option – c: $\left(-\frac{5F}{4}\hat{\imath}, 0\,\hat{\imath}, +\frac{5F}{4}\hat{\imath}\right)$

Charge, $q = q_1 = q_2 = q_3$

Distance between two charges = d

Now,

Force on the first charge due to charge q_3 will be

$$F_{12} = \frac{kq^2}{d^2}$$

Force on the first charge due to charge q_3 will be

$F_{13} = k\frac{q^2}{4d^2}$

Thus net force acting on charge q_1 is

$F_{net} = F_{12} + F_{13} = F + \frac{F}{4}$

$\therefore F_{net} = -\frac{5F}{4}\hat{\imath}$

Here, the negative sign shows that force will be along the negative x-axis.

From this, we can conclude that force acting on three charges are $\left(-\frac{5F}{4}\hat{\imath}, 0\,\hat{\imath}, +\frac{5F}{4}\hat{\imath}\right)$

29. Correct option – c: Both A and B

Given that,

According to the conclusion of A:

Charge flowing in 1 min, Q = 300 C

Time, t = 1 min = 60 sec

According to the conclusion of B:

Number of electron flows in 1 sec, $n = 3.125\times 10^{19}$ electron

Now,

Total current trough circuit according to A, $I_A = \frac{Q}{t} = 5$ A

Similarly

The total current through circuit according to B, $I_B = \frac{ne}{t} = \frac{3.125\times1.6\times10^{19}\times10^{-19}}{1} = 5$ A

30. Correct option – a: $10.6\times10^{-8}\Omega - m$

Given that,

$R_1 = 2\ \Omega$

$R_2 = 8\ \Omega$

$L_1 = L_2 = L$

$A_1 = A_2 = A$

Now,

$R = \frac{\rho L}{A} \Rightarrow \frac{R_1}{R_2} = \frac{\rho_1}{\rho_2}$

$\therefore \rho_2 = 2.65 \times 10^{-8} \times \frac{8}{2} = 10.6 \times 10^{-8}\Omega - \text{m}$

31. Correct option – c: $\frac{|E|}{|B|}$

According to the equation of Lorentz force, the net force acting on electrons due to the electric and magnetic field is given as

$\vec{F} = \vec{E} + (\vec{v} \times \vec{B})$

$\therefore |\vec{E}| + (|\vec{v}| \times |\vec{B}|) = 0$ ... ($\because$ Electron moves undeflected)

$|\vec{v}| = \frac{|E|}{|B|}$

32. Correct option – d: 0

Given that,

Charge, $q = 1.6 \times 10^{-19}$ C

Velocity, $\vec{v} = 4\hat{\imath} + 3\hat{k}$

Magnetic field, $\vec{B} = 3\hat{k} + 4\hat{\imath}$

Now the force acting on test charge is

$F_B = q(v \times B) = 0$ N

33. Correct option – a: $\frac{V}{R}$

At resonance, the value of current can be expressed as

$I = \frac{V}{R}$ ... ($\because X_c = X_L$)

34. Correct option – c: $\frac{50}{\pi}$Hz

Now,

As we know reactance of capacitor can be expressed as

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi X_c C}$

$f = \frac{1}{2\pi \times 1000 \times 10 \times 10^{-6}} = \frac{50}{\pi}$Hz

The frequency of an ac source for which a 10μF capacitor have a reactance of 1000 Ω is $\frac{50}{\pi}$Hz

35. Correct option – c:

36. Correct option – a: will be less if the length of wire is increased.

From Ohm's law, we know that,

$\frac{1}{R} = \frac{I}{V} = \text{slope}$

Hence for the given case, we can modify the above equation as

$\text{slope} = \frac{1}{R} = \frac{A}{\rho L}$

Now from this, we can conclude that the slope will be less if the length of the wire is increased.

37. Correct option – c: V

When a potential difference V is applied across a conductor at temperature T, the drift velocity of the electrons is proportional to V as we can see below.

$v_d = \frac{eV}{ml}\tau \ldots \left(\because E = \frac{V}{l}\right)$

$\therefore v_d \propto V$

38. Correct option – a: P and R only

From the given figure we can see that the net magnetic field at the centre of loop P and R will be zero since the current is flowing in a closed loop.

39. Correct option – d: $9B_0$

Now,

For the given case $2\pi r = 3\times 2\pi r' \Rightarrow r' = r/3$

Initial magnetic field

$\therefore B_0 = \frac{\mu_0 I}{2r} \Rightarrow B = \frac{9\mu_0 I}{2r} = 9B_0$

40. Correct option – d: An inductor can conduct in a dc circuit but not a capacitor.

As we know capacitors can be used for AC but not for DC, similarly, the inductor can conduct DC by obstructing AC.

41. Correct option – a: 53 V

The magnetic flux linked with a coil is given by ϕ

As we know according to Faraday's law of electromagnetic induction, the emf induced by the current-carrying coil is given as

$\epsilon = \frac{d\phi}{dt} = \frac{d(5t^2+3t+16)}{dt} = 10t + 3$

$\epsilon = 10(5) + 3 = 53\ V$

42. Correct option – d: Work is done by the external source

As we know the work is said to be done by an external source when a charge is moving against a coulomb's force of an electric field.

43. Correct option – d: Zero

A charge Q is at the centre of a circle with radius r. the work done in moving a test charge q_0 from point A to point B.

i.e., $W = Fs\cos\theta = 0$

44. Correct option – b: B

The magnetic field produced by a solenoid of N turns will be given as

$B = \mu_0 NI$

And for our case, if the number of turns is reduced to half and the current flowing through it doubled its initial value then the magnetic field produced by them will be

$B' = \mu_0 \left(\frac{N}{2}\right)(2I) = B$

45. Correct option – c: Assertion (A) is true, but Reason (R) is false.

For the given case, we know that a bar magnet experiences torque when it placed in a magnetic field

Also, we must note that the bar magnet doesn't exert a net torque on itself due to its own magnetic field although some of its components do.

46. Correct option – b: Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
For an LCR circuit that is connected to an AC power source the resonating frequency is given as
$\omega = \frac{1}{\sqrt{LC}} \ldots (X_c = X_L)$
Here, X_c and X_L are Capacitive reactance & Inductive reactance respectively.
Hence we can conclude that both Assertion and Reason are correct but Reason is not the correct explanation for assertion.

47. Correct option – a: Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
According to the equation of Lorentz force, the direction of the magnetic field is always perpendicular to the velocity of the particle and the force acting on it is expressed as.
$F_B = q(\vec{v} \times \vec{B})$ And from the above equation, we can see that force is also perpendicular to velocity or displacement.
Hence the force on the particle does not work because force is perpendicular to the displacement.
i.e., $W = F.dr = Fdr\cos\theta = 0$

48. Correct option – a: Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
For the given case, we know that resistance of the material is directly proportional to resistivity which is the intrinsic property of material, as a result of this resistance offered to the current flowing through the aluminium coil will be greater than the copper coil.
Hence we can conclude that the current in the copper coil is more than the aluminium coil although the induced emf is the same for both.

49. Correct option – a: Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
As we know, a transformer works on the principle of mutual inductance and it is used to increase or decrease AC voltage only.

Section C

50. Correct option – b: 2
For a uniform charged distribution the surface charged density for the sphere can be expressed as
$\sigma = \frac{q}{4\pi r^2} \Rightarrow \left(\frac{\sigma_A}{\sigma_B}\right) = \frac{q_A r_B^2}{q_B r_A^2}$
Now since both the spheres are connected, the electric potential across both surfaces will be
$V_A = V_B \Rightarrow \frac{q_A}{q_B} = \frac{r_A}{r_B}$

$\therefore \left(\frac{\sigma_A}{\sigma_B}\right) = \frac{r_B}{r_A} = 2:1$

51. Correct option – d: -F

As we know force on two parallel arms of the current-carrying square loop will be zero, since $\vec{dl} \times \vec{B} = 0$.

Thus the force on the arm opposite to the one on which force acting is F will be –F.

Hence option d is correct among all.

Case – study

52. Correct option – c: $\frac{r_2\epsilon_1+r_1\epsilon_2}{r_2+r_1}$

The equivalent emf of the given combination is given as shown below.

$$\epsilon_{eq} = \frac{\frac{\epsilon_1}{r_1}+\frac{\epsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{r_2\epsilon_1+r_1\epsilon_2}{r_2+r_1}$$

53. Correct option – b: $r_2\epsilon_1 + r_1\epsilon_2$

As we know for the given combination the equivalent emf is given as shown below.

$$\epsilon_{eq} = \frac{r_2\epsilon_1+r_1\epsilon_2}{r_2+r_1}$$

Hence from this, we can conclude that for the given combination of cells, the thermal B can be negative only if $r_2\epsilon_1 < r_1\epsilon_2$.

54. Correct option – a: $\frac{\epsilon_1+\epsilon_2}{r_1+r_2}$

According to Ohm's law, we know that electric current in a circuit is directly proportional to potential difference or emf across any given circuit.

i.e., $I = \frac{\epsilon_1+\epsilon_2}{r_1+r_2}$

55. Correct option – c: $\frac{r_1r_2}{r_1+r_2}$

In the given combination the cells are connected in parallel hence the equivalent internal resistance for the given circuit will be

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \Rightarrow r_{eq} = \frac{r_1r_2}{r_1+r_2}$$