

# Pragati Education Electric Charges & Field

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- 3] Quantization of electric charges  
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- 4] Conservation of charge.
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- h) Electric flux.
- i) Gauss Theorem.
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- l) electric field due to a uniformly charged infinite plane sheet.
- m) Field due to uniformly charged-thin spherical shell.

### Quick Notes

1. Electric charge → Intrinsic property of elementary particles of matter which give rise to electric force between various objects
    - \* It is a scalar quantity.
    - \* SI unit coulomb "C".
    - \* Smallest amount of charge  $e = 1.6 \times 10^{-19} C$ .
    - \* Two kind of charges.
      - Positive
      - Negative.
    - \* Like charges repel each other.
    - \* Unlike charges attract each other.
  2. Conductors → Substances through which electric charges can flow easily are called conductors e.g. metals.
  3. Insulators → Through which electric charges cannot flow easily e.g. plastic, wood.
    - \* The process in which a body shares its charges with the earth is called earthing or grounding.
- Basic Properties of electric charge
- 1) Additivity 2) Quantization 3) conservation.
  - 1) Additivity → Total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.

## Quantization of a Physical quantity

$$Q = ne$$

conservation \* Total charge of the system is conserved.  
\* charge can neither be created nor be destroyed.

\* electric charge vs Mass .

$$* m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Coulomb's law of electric force .

\* It states that the force of attraction or repulsion between two stationary point charges is directly proportional to  
(i) product of magnitudes of two charges  
(ii) Inversely proportional to square of the distance between them .

$$F \propto q_1 q_2$$

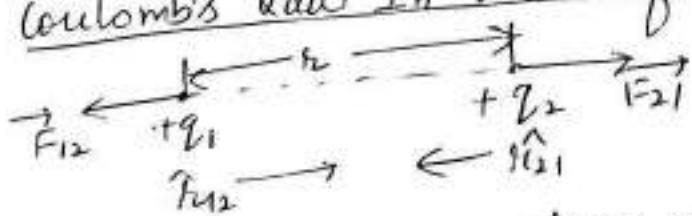
$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2} \rightarrow F = \frac{k q_1 q_2}{r^2}, \quad k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$\epsilon_0 = \frac{8.8551485 \times 10^{-12}}{9 \times 10^9 \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}}$$

\* 1 esu = 1 electrostatic unit of charge  
1 coulomb =  $3 \times 10^9$  statcoulombs .

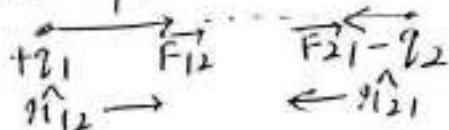
Coulomb's law In Vector form .



$$\vec{F}_{21} = \text{Force on charge } q_2 \text{ due to } q_1 \\ = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \hat{r}_{12} = \frac{\vec{r}_{12}}{r} \quad [\text{Unit vector in the direction } q_1 \rightarrow q_2]$$

$$\vec{F}_{12} = \text{Force on charge } q_1 \text{ due to } q_2 \\ = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad \hat{r}_{21} = \frac{\vec{r}_{21}}{r} \quad [\text{Unit vector in direction } q_2 \rightarrow q_1]$$

\* Coulombian forces b/w unlike charges ( $q_1 q_2 < 0$ ) are attractive



- \* Coulomb's law obey Newton's Third law of motion.
  - \* Central forces.
- Dielectric constant  $\rightarrow$  Force between two charges  $2q_1 q_2$  in vacuum.

$$F_{\text{vacuum}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad (1)$$

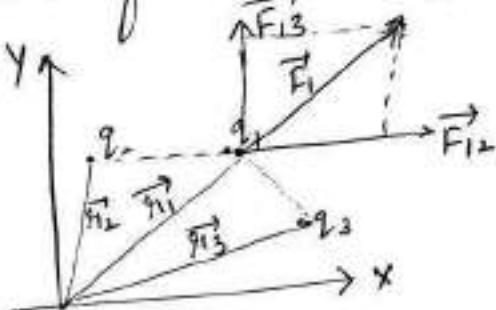
$$F_{\text{medium}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{Force between two charges in medium}$$

$$\frac{F_{\text{vac}}}{F_{\text{medium}}} = \frac{\epsilon_0}{\epsilon_r \epsilon_0} = K \quad \begin{matrix} \text{Divide (1) by (2)} \\ \downarrow \text{Dielectric constant} \end{matrix}$$

$$\begin{aligned} K_{\text{vacuum}} &= 1 \\ K_{(\text{air})} &= 1.00054 \\ K_{(\text{water})} &= 80 \end{aligned}$$

$$F_{\text{medium}} = F_{\text{vac}} \cdot \frac{K}{\epsilon_r}$$

The Superposition Principle  
 It states that when a no. of charges are interacting the total force on any charge is the vector sum of forces exerted on it due to all other charges. This force is unaffected by other forces.



$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1N}$$

$$\begin{aligned} \vec{F}_{12} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{11}-\vec{r}_{12}|^2} \frac{\vec{r}_{11}-\vec{r}_{12}}{|\vec{r}_{11}-\vec{r}_{12}|} \\ &= \frac{1}{4\pi\epsilon_0} q_1 q_2 \frac{\vec{r}_{11}-\vec{r}_{12}}{|\vec{r}_{11}-\vec{r}_{12}|^3} \end{aligned}$$

$$\begin{aligned} \text{Total force on charge } q_1 \text{ is} \\ \vec{F}_1 &= \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} + \dots + \frac{q_1 q_N}{r_{1N}^2} \hat{r}_{1N} \right] \end{aligned}$$

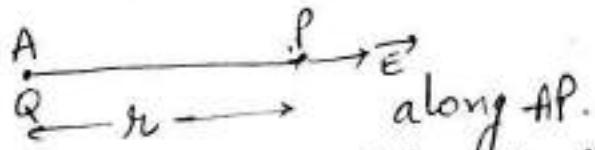
$$\vec{F}_1 = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^N \frac{q_i}{r_{1i}^2} \hat{r}_{1i}$$

# Pragati Education      Electric Field $\vec{E}$

- \* like a particle creates a gravitational field around it and this field exerts force on another particle placed in it.
- \* electric charge produces electric field in the space around it and then this electric field exerts a force on any charge (except on itself).
- \* Electric field takes finite time to propagate.
- \* If a charge is displaced from its position, the field at a distance "r" will change after time  $t = \frac{r}{c}$   $\rightarrow$  speed of light
- \*  $\vec{E} = \frac{\vec{F}}{q}$   $\rightarrow$  test charge (keeping magnitude very small).  
 [if magnitude is not very small, position of other charges may change]
- \* Electric field at point is a vector quantity.

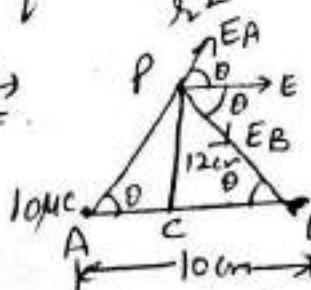
$$\vec{F} = K \frac{Qq}{r^2}$$

$$\frac{\vec{F}}{q} = \frac{KQ}{r^2} \rightarrow \vec{E}$$



SI Unit  $\rightarrow$  N/C Dimensional formula -  $MLT^{-3}A^{-1}$

Example  $\rightarrow$



Two charges  $+10\mu C$  and  $-10\mu C$  are placed at points A and B separated by a distance of 10cm. Find electric field

at a point P on the perpendicular bisector of AB at a distance of 12cm from its middle point.

$$AP = BP = \sqrt{(5)^2 + (12)^2} = 13\text{ cm}$$

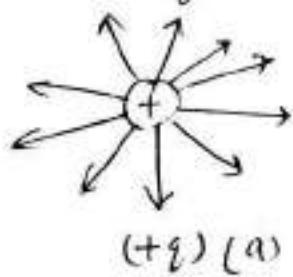
The field at P due to charge  $+10\mu C$  is

$$EA = \frac{10\mu C}{4\pi \epsilon_0 (13\text{ cm})^2} = 5.3 \times 10^6 \text{ N/C} \text{ along AP.}$$

$$EA = EB \quad E = EA \cos 60^\circ + EB \cos 60^\circ \\ = 4.1 \times 10^6 \text{ N/C}$$

## Lines of electric force

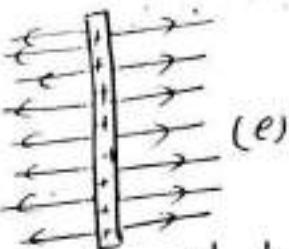
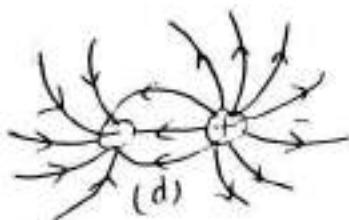
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(-q) (b)

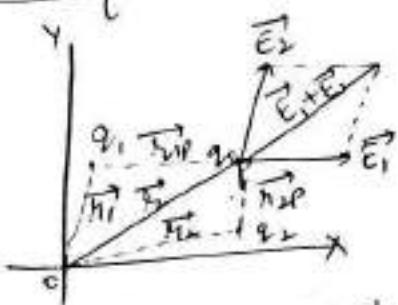


(c)



(e)

## Electric field due to system of point charges



Consider a system of particles (charges)  $q_1, q_2, \dots, q_N$  having position vectors  $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N$  with respect to origin.

$\vec{F}_i = \frac{1}{4\pi\epsilon_0} \frac{q_i q_0}{r_{ip}} \hat{n}_{ip}$ .  $\hat{n}_{ip}$  is a unit vector in the direction from  $q_i$  to  $P$  and  $r_{ip}$  is the distance between  $q_i$  and  $P$ .

$$\vec{E}_i = \frac{\vec{F}_i}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_{ip}} \hat{n}_{ip}$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2P}} \hat{n}_{2P} \quad [\text{Electric field at } P \text{ due to charge } q_2]$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_N$$

$$= \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1}{r_{1P}} \hat{n}_{1P} + \frac{q_2}{r_{2P}} \hat{n}_{2P} + \dots + \frac{q_N}{r_{NP}} \hat{n}_{NP} \right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{r_{ip}} \hat{n}_{ip}$$

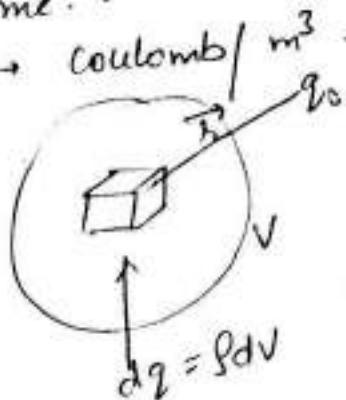
## Continuous charge distribution

(a) Volume charge distribution  $\rightarrow$  Charge distribution spread over a three dimensional volume.

$$\rho = \frac{q}{dV} \quad \text{SI unit} \rightarrow \text{coulomb/m}^3$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} \text{ c/m}^3$$

$$\vec{E}_V = \frac{\vec{F}_V}{q_0} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{r^2} dV \hat{r}$$



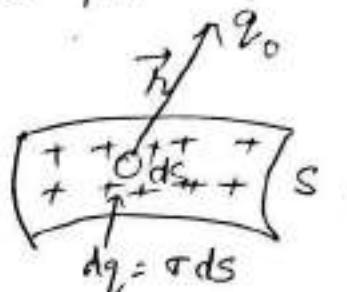
(b) Surface charge distribution

$$\sigma = \frac{q}{ds} \quad \text{SI Unit} = \text{c/m}^2$$

$$\sigma = \frac{q}{4\pi r^2}$$

$$dq = \sigma ds$$

$$\vec{F}_S = \frac{q_0}{4\pi\epsilon_0} \int_s \frac{\sigma}{r^2} ds \hat{r}$$



$$\vec{E}_S = \frac{\vec{F}_S}{q_0} = \frac{1}{4\pi\epsilon_0} \int_s \frac{\sigma}{r^2} ds \hat{r}$$

(c)

Line charge distribution

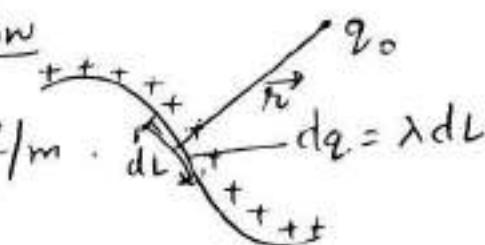
$$\lambda = \frac{dq}{dL} \rightarrow \lambda = \frac{q}{2\pi R} \text{ c/m}$$

$$dq = \lambda dL$$

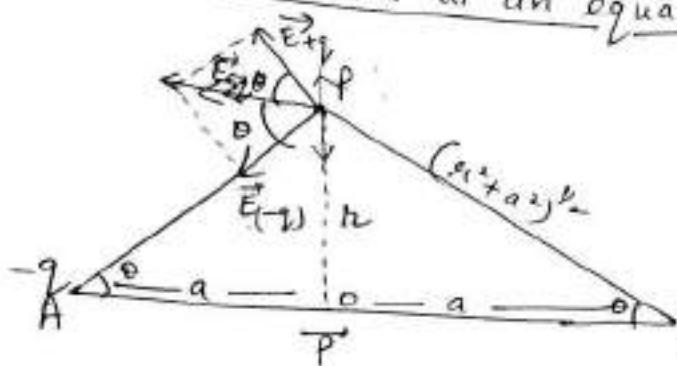
$$\vec{F}_L = \frac{q_0}{4\pi\epsilon_0} \int_L \frac{\lambda}{r^2} dL \hat{r}$$

$$\vec{E}_L = \frac{\vec{F}_L}{q_0} = \frac{1}{4\pi\epsilon_0} \int_L \frac{\lambda}{r^2} dL \hat{r}$$

$$\vec{E}_{\text{cont}} = \vec{E}_V + \vec{E}_S + \vec{E}_L$$



## Electric Field at an equatorial point of a Dipole



$$E_{+2} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \text{ [Along BP]}$$

$$E_{-2} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \text{ [Along PA]}.$$

$$B^2 \quad E_{+2} = E_{-2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2}{r^2+a^2}$$

\* Components of  $\vec{E}_{-2}$  &  $\vec{E}_{+2}$  normal to dipole axis will cancel out. Components parallel to dipole axis add up. So Total electric field  $\vec{E}_{\text{eq}}$  is opposite to  $\vec{p}$ .

$$\vec{E}_{\text{eq}} = -(E_{-2} \cos\theta + E_{+2} \cos\theta) \hat{p}$$

$$= -2E_{-2} \cos\theta \hat{p} \quad [E_{-2} = E_{+2}]$$

$$= -\frac{2\lambda}{4\pi\epsilon_0} \frac{1}{r^2+a^2} \frac{a}{\sqrt{r^2+a^2}} \hat{p} \quad [\cos\theta = \frac{a}{\sqrt{r^2+a^2}}]$$

$$E_{\text{eq}} = \frac{-1}{4\pi\epsilon_0} \cdot \frac{\vec{p}}{(r^2+a^2)^{3/2}} \hat{p}$$

$$\text{if } r \gg a, \vec{E}_{\text{eq}} = \frac{-1}{4\pi\epsilon_0} \cdot \frac{\vec{p}}{r^3} \hat{p}$$

Direction of  $\vec{E}$  at any point on the equatorial line of dipole will be antiparallel to dipole moment  $\vec{p}$ .

Relation b/w Axial &  $E_{\text{eq}}$ .

$$\frac{\text{Axial}}{E_{\text{eq}}} = \frac{K2p}{\lambda^3 \times Kp} \times \frac{r^3}{r^3} = 2$$

$$\text{Axial} = 2 E_{\text{eq}}$$

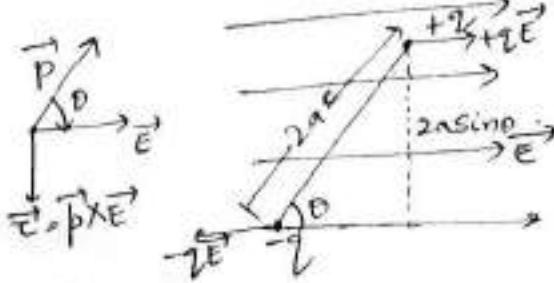
## Toque on a dipole in a Uniform Electric field

$$\vec{F}_{\text{Total}} = +q\vec{E} - q\vec{E} = 0$$

$$\tau = qE \times 2a \sin\theta$$

$$= (q \times 2a) E \sin\theta$$

$$\boxed{\tau = p E \sin\theta}$$



## Electric Dipole

A pair of equal and opposite charges separated by a small distance is called electric dipole.

Dipole moment  $\vec{P} = q \times 2a$  Force on +q

Direction is along the dipole axis from negative to positive charge.  $+q$

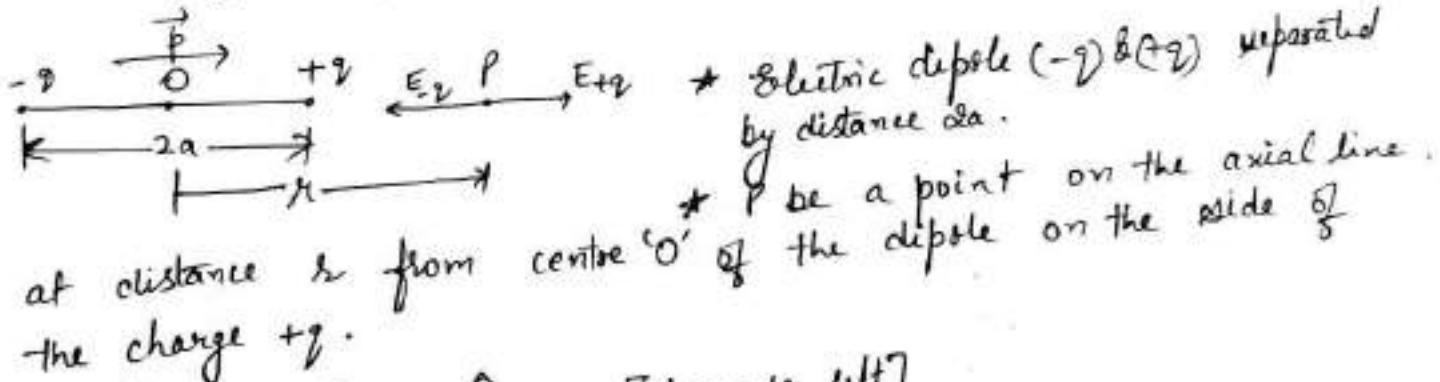
SI unit - coulombmeter. e.g.  $H_2O, HCl$   $e = -$

Dipole field → The electric field produced by an electric dipole is called dipole field. field  
bale  
let to

## Variation of Dipole field with distance

- \* Total charge of an electric dipole is zero.
- \* But  $E$  of an electric dipole is not zero.
- \* Because charges  $+q$  and  $-q$  are separated by distance, so  $E$  due to them when added cancel out.

## Electric field at axial point of a dipole



$$\vec{E}_{-q} = \frac{-q}{4\pi\epsilon_0(r+a)^2} \hat{p} \quad [\text{towards left}]$$

$$\vec{E}_{+q} = \frac{+q}{4\pi\epsilon_0(r-a)^2} \hat{p} \quad [\text{towards right}]$$

Resultant  $\vec{E}$  at point  $P$  is.

$$\vec{E}_{\text{axial}} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{p}$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{4ar}{(r^2-a^2)^2} \hat{p}$$

$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2-a^2)^2} \hat{p}$$

if  $r \gg a$ ,  $a^2$  can be neglected.

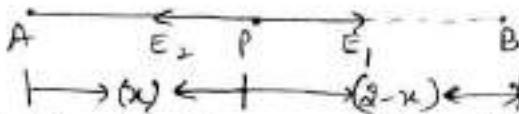
$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \hat{p} \quad (\text{towards right})$$

\* Direction of  $\vec{p}$  ve to +ve

## Important Questions of Superposition principle of Electric field :-

Q1) Two point charges of  $+5 \times 10^{-9} C$  and  $+20 \times 10^{-9} C$  are separated by a distance of 2m. Find the point on the line joining them at which electric field intensity is zero.

Ans:-



Electric field at point P will be zero, if

$$E_1 = E_2$$

$$\frac{1}{4\pi\epsilon_0} \frac{5 \times 10^{-9}}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{20 \times 10^{-9}}{(2-x)^2}$$

$$x = \frac{2}{3} m \text{ or } -2 m.$$

At  $x = -2 m$  i.e. at 2m left of  $E_1$ , electric fields due to both charges will be in ~~opposite direction~~ same direction. So  $x = -2 m$  is not a possible solution.

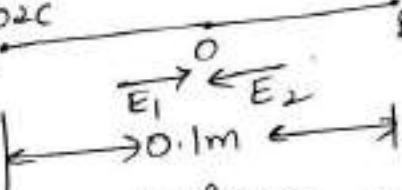
So  $E$  will be zero at  $\frac{2}{3} m$  to the right of  $E_1$ .

Q2) Two point charges  $q_1 = +0.2 C$  and  $q_2 = +0.4 C$  are placed

0.1m apart. Calculate the electric field at

- a) mid point between the charges.
- b) a point on the line joining  $q_1$  and  $q_2$  such that it is 0.05m away from  $q_2$  and 0.15m away from  $q_1$ .

Ans:-  $q_1 = +0.2 C$        $q_2 = +0.4 C$



$$E_1 = \frac{kq_1}{r_{12}^2} = \frac{9 \times 10^9 \times 0.2}{(0.05)^2} = 7.2 \times 10^9 N/C. \text{ [acting along AO]}$$

$$E_2 = \frac{kq_2}{r_{12}^2} = \frac{9 \times 10^9 \times 0.4}{(0.05)^2} = 14.4 \times 10^9 N/C. \text{ [acting along BO]}$$

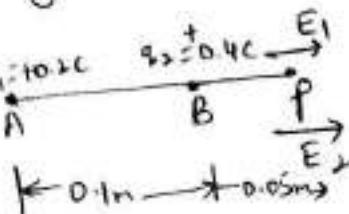
(b) Electric field at P due to  $q_1$

$$E_1 = \frac{kq_1}{r_{12}^2} = \frac{9 \times 10^9 \times 0.2}{(0.15)^2}$$

Electric field at P due to  $q_2$ .

$$E_2 = \frac{kq_2}{r_{12}^2} = \frac{9 \times 10^9 \times 0.4}{(0.05)^2} \text{ acting along BP.}$$

$$\text{so Net } E = E_1 + E_2 = 1.52 \times 10^{12} \text{ acting.}$$

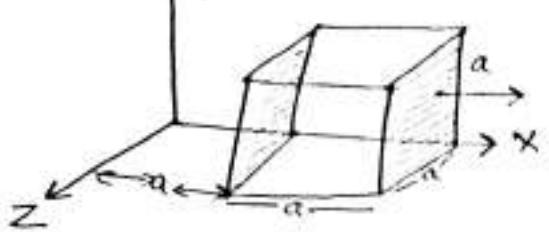


Pragati Education Flux & its numericals

$$\phi = \vec{E} \cdot \vec{S}$$

$$\phi = \frac{q}{\epsilon_0}$$

1. flux density =  $\frac{\phi_E}{S}$  N/C



Electric field are  $E_x = \alpha x^{\nu_2}$ ,  $E_y = E_z = 0$  in which  $\alpha = 800 \text{ N/Cm}^2$ . Calculate  
 (i) the flux  $\phi_E$  through the cube.  
 (ii) charge within cube  $a=0.1\text{m}$ .

Ans: (ii) As  $\vec{E}$  is along x-axis.

Magnitude of  $\vec{E}$  at left face is

$$E_L = \alpha x^{\nu_2} = \alpha a^{\nu_2} \quad [x=a \text{ at left face}]$$

$$\phi_L = \vec{E}_L \cdot \vec{DS} = E_L \Delta S \cos 0^\circ \\ = E_L a^2 \cos 180^\circ = -E_L a^2 \quad [\theta = 180^\circ \text{ for left face}]$$

$$E_R = \alpha x^{\nu_2} = \alpha (2a)^{\nu_2} \quad [x=2a \text{ at right face}]$$

$$\phi_R = E_R \Delta S \cos 0^\circ = E_R a^2 \quad [\theta = 0^\circ \text{ for right face}]$$

flux through cube.

$$\phi_E = \phi_L + \phi_R = E_R a^2 - E_L a^2 = a^2 (E_R - E_L) \\ = \alpha a^2 [(2a)^{\nu_2} - a^{\nu_2}] \\ = \alpha a^{5/2} [\sqrt{2} - 1]$$

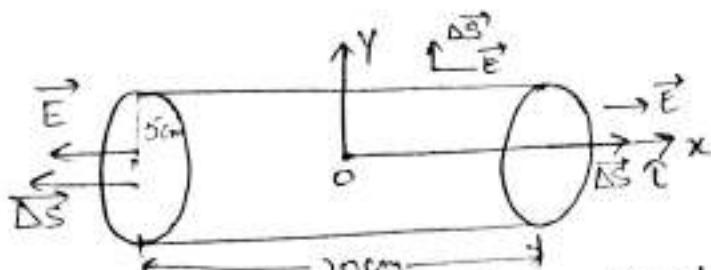
$$= 800 (0.1)^{5/2} (\sqrt{2} - 1) = 1.05 \text{ Nm}^2 \text{C}^{-1}$$

(iii) By Gauss's theorem, total charge inside the cube is .

$$q = \epsilon_0 \phi_E = \frac{1}{4\pi \times 9 \times 10^9} \times 1.05 = 9.27 \times 10^{-12} \text{ C}.$$

2. An electric field is uniform and in the positive x direction for positive x and uniform with the same magnitude in negative x.  $\vec{E} = 200 \text{ i N/C}$  for  $x > 0$   
 $\vec{E} = -200 \text{ i N/C}$  for  $x < 0$

A right circular cylinder of length 20cm and radius 5cm has its centre at the origin and its axis along x axis so that one face is at  $x = +10\text{cm}$  & other is at  $x = -10\text{cm}$ . (a) What is net outward flux through each face? (b) flux through side of cylinder.  
 (c) net outward flux through cylinder.  
 (d) net charge inside the cylinder.



(i) On left face:  $\vec{E} = -200 \hat{i} \text{ N/C}$   
 $\vec{dS} = -\Delta S \hat{i} = -\pi (0.05)^2 \hat{i} \text{ m}^2$

$$\Phi_E = \vec{E} \cdot \vec{dS}$$

$$= +200 \times \pi (0.05)^2 \hat{i} \text{ Nm}^2 \text{ C}^{-1}$$

$$= +1.57 \text{ Nm}^2 \text{ C}^{-1}$$

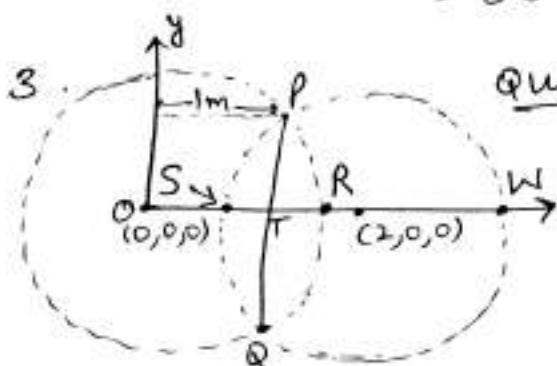
(ii) On right face  
 $\vec{E} = 200 \hat{i} \text{ N/C}$ .  $\vec{dS} = \Delta S \hat{i} = \pi (0.05)^2 \hat{i} \text{ m}^2$

$$\Phi_E = \vec{E} \cdot \vec{dS} = +1.57 \text{ Nm}^2 \text{ C}^{-1}$$

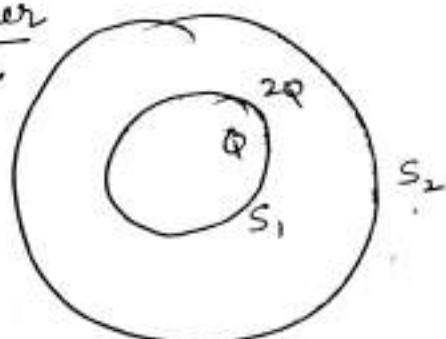
(iii) through side of cylinder  $\Phi_E = \vec{E} \cdot \vec{dS} = E dS \cos 90^\circ = 0$

(iv) Net flux through cylinder  $\Phi_E = 1.57 + 1.57 + 0$   
 $= 3.14 \text{ Nm}^2 \text{ C}^{-1}$ .

Net charge  $q = \epsilon_0 \Phi_E$   
 $= 8.85 \times 10^{-12} \times 3.14 = 2.78 \times 10^{-11} \text{ C}$ .



Questions to Ponder

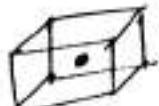


i.  $\frac{\phi_1}{\phi_2} = \frac{1/60}{3Q/60} = 1:3$ .

ii)  $\phi = \frac{Q}{k\epsilon_0}$ .

If dielectric constant  $k$  is introduced in it ( $S_1$ )

$= \frac{Q}{5\epsilon_0}$ .  $\left[ E' = \frac{E}{k} \right]$ .



$Q:Q_2$